KISHORE VAIGYANIK PROTSAHAN YOJANA – 2020-21

STREAM – SX

Date : 31/01/2021 | **Time : 3 hours** | **Maximum Marks: 160**

INSTRUCTIONS

Read the following instructions carefully before you open the question booklet.

The question paper consists of two parts (both contain only multiple choice questions) for 160 marks. There will be four sections in Part I (each section containing 20 questions) and four sections in Part II (each section containing 10 questions)

PART-I

(i) There are 80 objective type questions. **20 questions** from each subject (**Mathematics**, **Physics**, **Chemistry** & **Biology**), you have to attempt any 3 subjects out of 4 subjects.

(ii) Each correct answer gets 1 mark and for each incorrect answer 0.25 mark will be deducted.

Part-II

(i) There are 40 objective type questions. **10 questions** from each subject (**Mathematics**, **Physics**, **Chemistry** & **Biology**), you have to attempt 2 subjects out of 4 subjects.

(ii) Each correct answer gets **2 marks** and for each incorrect answer **0.5 mark** will be deducted.

Name of Student :..

Batch : ....

Enrolment No. | | | | | | | | | | | | | | | | |

KISHORE VAIGYANIK PROTSAHAN YOJANA (KVPY) 2020-21

Date of Examination – 31st January, 2021

SOLUTIONS

PART-I : MATHEMATICS

[Q.1] Consider the following statements:

i. $\lim_{n\to\infty} \frac{2^n + (-2)^n}{2^n}$ does not exist

ii. $\lim_{n\to\infty} \frac{3^n + (-3)^n}{4^n}$ dose not exist

Then

[A] i is true and ii is false

[B] i is false and ii is

[C] i and ii are true

[D] Neither I nor ii is true

[ANS] **A**

[SOL] i. $\lim_{n\to\infty} \frac{2^n + (-2)^n}{2^n} = \lim_{n\to\infty} 1 + (-1)^n = DNE$

ii. $\lim_{n\to\infty} \frac{3^n + (-3)^n}{4^n} = \lim_{n\to\infty} \left(\frac{3}{4}\right)^n + \left(\frac{-3}{4}\right)^n = 0 + 0 = 0$

[Q.2] Consider a regular 10-gon with its vertices on the unit circle. With one vertex fixed, draw straight lines to the other 9 vertices. Call them L_1, L_2, …, L_9 and denote their lengths by $\ell_1, \ell_2, ..., \ell_9$ respectively. Then the product $\ell_1, \ell_2, ..., \ell_9$ is

[A] 10

[B] $10\sqrt{3}$

[C] $\frac{50}{\sqrt{3}}$

[D] 20

[ANS] **A**

[SOL] $\ell_1 = 2\sin\frac{\pi}{10} = \ell_9$

$\ell_2 = 2\sin\frac{2\pi}{10} = \ell_8$

$\ell_3 = 2\sin\frac{3\pi}{10} = 2\cos\frac{2\pi}{10}\ell_7$

$\ell_4 = 2\sin\frac{4\pi}{10} = 2\cos\frac{\pi}{10}\ell_6$

$$\ell_5 = 2\sin\frac{5\pi}{10} = 2$$

$$\ell_1 \times \ell_2 ... \ell_9 = 2\left(16\sin\frac{\pi}{10}\sin\frac{2\pi}{10}\cos\frac{2\pi}{10}\cos\frac{\pi}{10}\right)^2$$

$$= 2\left(4\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\right)^2 = 2\times 1\left(\frac{4\sqrt{10-2\sqrt{5}}}{4}\times\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2$$

$$= 2\times\frac{80}{16} = 10$$

[Q.3] The value of the integral $\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+e^x}dx$ is

[A] $\frac{\pi}{6}$

[B] $\frac{\pi}{4}$

[C] $\frac{\pi}{2}$

[D] $\frac{\pi^2}{2}$

[ANS] **B**

[SOL] $\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+e^x}dx = \int_0^{\pi/2}\left(\frac{\sin^2 x}{1+e^x}+\frac{\sin^2(-x)}{1+e^{-x}}\right)dx$

$$= \int_0^{\pi/2}\left(\frac{\sin^2 x}{1+e^x}+\frac{e^x\sin^2 x}{1+e^{-x}}\right)dx$$

$$= \int_0^{\pi/2}\sin^2 x\, dx = \frac{\pi}{4}$$

[Q.4] Let $\mathbb{R}$ be the set of all real numbers and f(x) = $\sin^{10} x(\cos^8 x + \cos^4 x + \cos^2 x + 1)$ for $x \in \mathbb{R}$. Let S = $\{\lambda \in \mathbb{R} \mid$ there exists a point $c \in (0, 2\pi)$ with $f'(c) = \lambda f(c)\}$.

Then

[A] $S = \mathbb{R}$

[B] S = {0}

[C] $S = [0, 2\pi]$

[D] S is a finite set having more than one element

[ANS] A

[SOL] $\ln f(x) = 10 \ln \sin x - \ln (1+ \cos^2 x + \cos^4 x + \cos^8 x)$

$$\Rightarrow \frac{f'(x)}{f(x)} = 10\cot x - \sin 2x\left[\frac{1+2\cos^2 x+4\cos^6 x}{1+\cos^2 x+\cos^4 x+\cos^8 x}\right]$$

$\because$ f(x) is periodic with period π and $\frac{f'(x)}{f(x)}$ is continuous in $(0,\pi)$

$$\frac{f'(x)}{f(x)} = \underbrace{10\cot x}_{\substack{\text{Range si} \\ (-\infty,\infty)\text{ for} \\ x\in(0,\pi)}} - \underbrace{\sin 2x\left[\frac{1+2\cos^2 x+4\cos^6 x}{1+\cos^2 x+\cos^4 x+\cos^8 x}\right]}_{\text{finite number for } x\in(0,\pi)}$$

So $\frac{f'(x)}{f(x)}$ has range $(-\infty, \infty)$

Hence $\lambda \in R$

[Q.5] A person standing on the top of a building of height $60\sqrt{3}$ feet observed the top of a tower to lie at an elevation of 45°. That person descended to the bottom of the building and found that the top of the same tower is now at an angle of elevation of 60°. The height of the tower (in feet) is

[A] 30

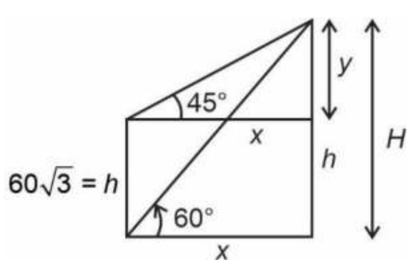
[B] $30\left(\sqrt{3}+1\right)$

[C] $90\left(\sqrt{3}+1\right)$

[D] $150\left(\sqrt{3}+1\right)$

[ANS] C

[SOL]



$y = x\tan 45^\circ = x$

$H = x\tan 60^\circ = \sqrt{3}x$

$h = H - y$

$60\sqrt{3} = (\sqrt{3}-1)x$

$x = 30\sqrt{3}(\sqrt{3}+1)$

$H = \sqrt{3}x = 90\left(\sqrt{3}+1\right)$

[Q.6] Assume that $3.13 \le \pi \le 3.15$. The integer closest to the value of $\sin^{-1}$ (sin1 cos4 + cos1 sin4), where 1 and 4 appearing in sin and cos are given in radians, is

[A] – 1

[B] 1

[C] 3

[D] 5

[ANS] **A**

[SOL] $\sin^{-1}(\sin 1 \cos 4 + \cos 1 \sin 4)$

$= \sin^{-1}(\sin 5)$

$= 5 - 2\pi = 5 - 2(3.14) = -1.28$

i.e., nearest integer is –1

[Q.7] The maximum value of the function f(x) = e^x + xlnx on the interval $1 \le x \le 2$ is

[A] $e^2 + in2 + 1$

[B] $e^2 + 2\ \ln 2$

[C] $e^{\pi/2} + \frac{\pi}{2}\ln\frac{\pi}{2}$

[D] $e^{3/2} + \frac{3}{2}\ln\frac{3}{2}$

[ANS] **B**

[SOL] $f(x) = e^x + x\ \ln x$

$f'(x) = e^x + 1 + \ln x \qquad > 0 \qquad \forall\ x \in (1, 2)$

i.e., f(x) is increasing hence $f(x)_{max}$ at x = 2

$f(2) = e^2 + 2\ \ln 2$

[Q.8] Let A be a 2 × 2 matrix of the form $A = \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}$, where a, b are integers and $-50 \le b \le 50$. The number of such matrices A such that A^{-1}, the inverse of A, exists and A^{-1} contains only integer entries is

[A] 101

[B] 200

[C] 202

[D] 101^2

[ANS] **C**

[SOL] $a-b\neq 0$

$$A^{-1}=\begin{bmatrix}\frac{1}{a-b} & \frac{b}{a-b}\\ -\frac{1}{a-b} & \frac{a}{a-b}\end{bmatrix}$$

For all entries to be integer

$a-b=\pm 1$

$b\in[-50, 50]\to 101$ values

Corresponding a has 202 values

Hence 202 such matrices possible

[Q.9] Let $A=(a_{ij})_{1\le i,\, j\le 3}$ be a 3 × 3 invertible matrix where each a_{ij} is a real number. Denote the inverse of the matrix A by A^{-1}. If $\sum_{j=1}^{3} a_{ij}=1$ for $1\le i\le 3$, then

[A] Sum of the diagonal entries of A is 1

[B] Sum of each row of A^{-1} is 1

[C] Sum of each row and each column of A^{-1} is 1

[D] Sum of the diagonal entries of A^{-1} is 1

[ANS] **B**

[SOL] $\because$ $A\cdot A^{-1}=I$

$$\Rightarrow \begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}\begin{bmatrix}x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\\ z_1 & z_2 & z_3\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{cases}x_1a_1+y_1a_2+z_1a_3=1\\ x_2a_1+y_2a_2+z_2a_3=0\\ x_3a_1+y_3a_2+z_3a_3=0\end{cases}$$

On adding we get, $(\sum x_i)a_1+(\sum y_i)a_2+(\sum z_i)a_3=1$

And it is given that $a_1 + a_2 + a_3 = 1$

So $\sum x_i=1=\sum y_i=\sum z_i$

[Q.10] Let x, y be real numbers such that $x > 2y > 0$ and $2\log(x - 2y) = \log x + \log y$. Then the possible value(s) of $\frac{x}{y}$

[A] is 1 only

[B] are 1 and 4

[C] is 4 only

[D] is 8 only

[ANS] **C**

[SOL] $\log(x-2y)^2 = \log xy$

$\Rightarrow \quad x^2 + 4y^2 - 4xy = xy$

$\Rightarrow \quad x^2 + 4y^2 - 5xy = 0$

$\Rightarrow \quad \left(\frac{x}{y}\right)^2 - 5\left(\frac{x}{y}\right) + 4 = 0$

$\Rightarrow \quad \left(\frac{x}{y} - 1\right)\left(\frac{x}{y} - 4\right) = 0$

$\Rightarrow \quad \frac{x}{y} = 1$ or 4 but if $\frac{x}{y} = 1 \;\; x - 2y < 0$

Hence only $\frac{x}{t} = 4$

[Q.11] Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\,(b < a)$, be a ellipse with major axis AB and minor axis CD. Let F_1 and F_2 be its its two foci, with A, F_1, F_2, B in that order on the segment AB. Suppose $\angle F_1CB = 90°$. The eccentricity of the ellipse is

[A] $\frac{\sqrt{3}-1}{2}$

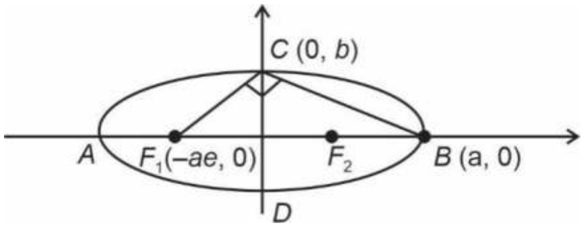
[B] $\frac{1}{\sqrt{3}}$

[C] $\frac{\sqrt{5}-1}{2}$

[D] $\frac{1}{\sqrt{5}}$

[ANS] **C**

[SOL]



$\therefore \quad \angle F_1CB = 90^\circ$

$\Rightarrow \quad \frac{-b}{a}\cdot\frac{b}{ae} = -1$

$\Rightarrow \quad \frac{b^2}{a^2} = e \Rightarrow e = 1 - e^2$

$\Rightarrow \quad e^2 + e - 1 = 0$

$\Rightarrow \quad e = \frac{-1 \pm \sqrt{5}}{2}$

$\Rightarrow \quad e = \frac{\sqrt{5} - 1}{2}$

[Q.12] Let A denote the se5 or all real numbers x such that $x^3 - [x]^3 = (x - [x])^3$. Where [x] is the greatest integer less than or equal to x. Then

[A] A is a discrete set of a least two points

[B] A contains an interval, but is not an interval

[C] A is an interval, but a proper subset of $(-\infty, \infty)$

[D] $A = (-\infty, \infty)$

[ANS] **B**

[SOL] $x^3 - [x]^3 = (x - [x])^3$

$\Rightarrow \quad (x - [x])(x^2 + [x]^2 + x[x]) = (x - [x])^3$

$\Rightarrow \quad x - [x] = 0 \quad$ or $\quad x[x] = 0$

$\Rightarrow \quad \{x\} = 0 \quad$ or $\quad x[x] = 0$

$\Rightarrow \quad x \in Z \qquad x \in [0, 1)$

Hence solution in $x \in [0, 1) \cup Z$

[Q.13] Define a sequence $\{s_n\}$ of real number by

$$s_n = \sum_{k=0}^{n} \frac{1}{\sqrt{n^2 + k}}, \text{ for } n \geq 1.$$

Then $\lim\limits_{n\to\infty} s_n$

[A] Does not exist

[B] Exists and lies in the interval (0, 1)

[C] Exists and lies in the interval [1, 2)

[D] Exists and lies in the interval [2, ∞)

[ANS] **C**

[SOL] $s_n = \sum_{k=0}^{n} \frac{1}{\sqrt{n^2+k}}$

$$\frac{1}{\sqrt{n^2+n}} + \frac{1}{\sqrt{n^2+n}} + \ldots + \frac{1}{\sqrt{n^2+n}} \le \frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2+1}} + \ldots + \frac{1}{\sqrt{n^2+n}} \le \frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2}} + \ldots + \frac{1}{\sqrt{n^2}}$$

$$\lim_{n\to\infty} \frac{n+1}{\sqrt{n^2+n}} \le \lim_{n\to\infty} s_n \le \lim_{n\to\infty} \frac{n+1}{\sqrt{n^2}}$$

$$1 \le \lim_{n\to\infty} s_n \le 1$$

$$\lim_{n\to\infty} s_n = 1$$

[Q.14] Let $\mathbb{R}$ be the set of all real number and $f:\mathbb{R}\to\mathbb{R}$ be a continuous function. Suppose $|f(x)-f(y)| \ge |x-y|$ for all real number x and y. Then

[A] f is one-one, need not be onto

[B] f is onto, but need not be one-one

[C] f need not be either one-one or onto

[D] f is one-one and onto

[ANS] **D**

[SOL] f(x) cannot be many one.

$\because$ If f(x) is many one, then there exist x_1 and x_2 such that $f(x_1) = f(x_2)$ but $x_1 \ne x_2$.

Then $0 \ge |x_1 - x_2|$, which is a contradiction

$\Rightarrow$ f(x) is one-one.

Given $|f(x)-f(y)| \ge |x-y|$ .....(1)

At any point x = a

$$\lim_{h\to 0} \frac{|f(a+h)-f(a)|}{|(a+h)-a|} \ge 1$$

$\Rightarrow$ $|f'(a)| \ge 1$

$f'(a) \ge 1$ or $r'(a) \le -1$

But both cannot hold simultaneously ($\because f(x)$ is one-one, proved abve)

Also |x – y| can assume very large value and $|f(x)| - f(y) \geq |x - y|$

$\Rightarrow$ f(x) is onto

Case –I :

$f'(a) \geq 1$ and WLOG $x < y \Rightarrow f(x) < f(y)$

Let y = k = finite

$|(f(x) - f(y))| \geq |x - y|$

$-(f(x) - f(y)) \geq -(x - y)$

$f(y) - f(x) \geq y - x$

$f(k) - f(x) \geq k - x$

$f(k) \leq f(k) - k + x$

When $x \to -\infty$

$f(x) \to -\infty$

Case – II :

$f'(a) \leq -1$

Let y = k = finite ; WLOG x > k, f(x) < f(k)

$|f(x) - f(k)| \geq |x - k|$

$-(f(x) - f(k)) \geq x - k$

$f(x) \leq f(k) + k - x$

When $x \to \infty, f(x) \leq f(k) + k - \infty$

Then $f(x) \to -\infty$

[Q.15] Let $f(x) = \begin{cases} \dfrac{x}{\sin x}, & x \in (0, 1) \\ 1, & x = 0 \end{cases}$

Consider the integral $I_n = \sqrt{n} \int_0^{1/n} f(x) e^{-nx} dx$.

Then $\lim_{n \to \infty} I_n$

[A] Does not exist

[B] Exists and is 0

[C] Exists and is 1

[D] Exists and is $1 - e^{-1}$

[ANS] **B**

[SOL] Let $n = \frac{1}{m}$

So, $\lim_{m\to 0} I_n = \lim_{m\to 0} \frac{\int_0^m \frac{x \cdot e^{\frac{x}{m}}}{\sin x} dx}{\sqrt{3}}$ Let x = mt

$\Rightarrow \lim_{m\to 0} I_n = \lim_{m\to 0} \frac{\int_0^1 \frac{m^2 \cdot e^{-1} \cdot 1}{\sin(mt)} dx}{\sqrt{m}}$ dx = mdt

$= \lim_{m\to 0} \sqrt{m} \int_0^1 \left(\frac{mt}{\sin(mt)}\right) \cdot e^{-1} dt$

$= \lim_{m\to 0} \sqrt{m} \int_0^1 e^{-1} dt$

$= \lim_{m\to 0} \sqrt{m}(1 - e^{-1}) = 0$

[Q.16] The value of the integral $\int_1^3 \left((x-2)^4 \sin^3(x-2) + (x-2)^{2019} + 1\right) dx$ is

[A] 0

[B] 2

[C] 4

[D] 5

[ANS] **B**

[SOL] $I = \int_1^3 \left((x-2)^4 \sin^3(x-2) + (x-2)^{2019} + 1\right) dx$ .....(1)

Use of formula $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\therefore \quad I = \int_1^3 \left\{(4-x-2)^4 \sin^3(4-x-2) + (4-x-2)^{2019} + 1\right\} dx$

$\therefore \quad I = \int_1^3 \left\{(-x-2)^4 \sin^3(x-2) - (x-2)^{2019} + 1\right\} dx$ .....(2)

From equation (1) + equation (2) we get;

$2I = \int_1^3 2dx$

$\Rightarrow \quad I = \int_1^3 dx = 2$

[Q.17] In a regular 15-sided polygon with all its diagnonals drawn, a diagonal is chosen at random. The probability that it is neither a shortest diagonal nor a longest diagonal is

[A] $\frac{2}{3}$

[B] $\frac{5}{6}$

[C] $\frac{8}{9}$

[D] $\frac{9}{10}$

[ANS] **A**

[SOL] Total number of diagonals of 15 sided polygons $= {}^{15}C_2 - 15 = \frac{15 \times 14}{2} - 15 = 90$

$\therefore$ Number of total shortest diagonals = 15

And number of longest diagonals = 15

$\therefore$ The probability that the selected diagonal is neither shortest nor longest

$= \frac{90 - 30}{90} = \frac{60}{90} = \frac{2}{3}$

[Q.18] Let M = $2^{30} - 2^{15} + 1$, and M^2 be expressed in base 2. The number of 1's in this base 2representation of M^2 is

[A] 29

[B] 30

[C] 59

[D] 60

[ANS] **B**

[SOL] $M^2 = 2^{60} - 2^{46} + 2^{32} + 2^{30} - 2^{16} + 1$

$\Rightarrow \quad M^2 = 2^{46}\left[\frac{2^{14} - 1}{2 - 1}\right] + 2^{32} + 2^{16}\left[\frac{2^{14} - 1}{2 - 1}\right] + 1$

$\Rightarrow \quad M^2 = 2^{46}[1 + 2 + 2^2 + \ldots + 2^{13}] + 2^{32} + 2^{16}[1 + 2 + 2^2 + \ldots + 2^{13}] + 1$

$\Rightarrow \quad M^2 = \underbrace{2^{59} + 2^{58} + \ldots + 2^{46}}_{\text{14 terms}} + 2^{32} + \underbrace{2^{29} + 2^{28} + \ldots + 2^{16}}_{\text{14 terms}} + 2^0$

So in base 2 representation of M^2, there will be 30 times digit 1.

[Q.19] Let ABC be a triangle such that AB = 15 and AC = 9. The bisector of $\angle BAC$ meets BC in D. If $\angle ACB = 2\angle ABC,$ then BD is

[A] 8

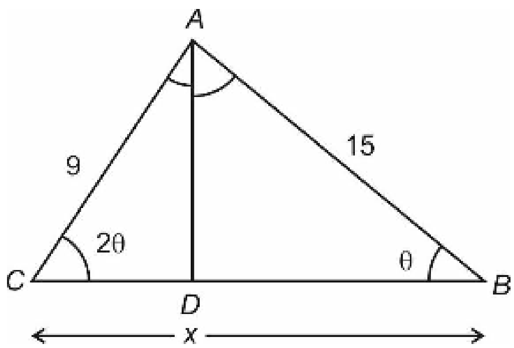
[B] 9

[C] 10

[D] 12

[ANS] **C**

[SOL]



Let $\angle ABC = \theta$ than $\angle ACB = 2\theta,$ and BC = x

By sine rule in ΔABC

$$\frac{\sin\theta}{9} = \frac{\sin 2\theta}{15} = \frac{\sin 3\theta}{x}$$

$$\Rightarrow \quad \frac{1}{9} = \frac{2\cos\theta}{15} = \frac{3-4\sin^2\theta}{x},$$

$\because \quad \sin\theta \neq 0.$

$\therefore \quad \cos\theta = \frac{5}{6}$ hence $\sin^2\theta = 1 - \frac{25}{36} = \frac{11}{36}.$

$\therefore \quad x = 9(3 - 4\sin^2\theta)$

$$= 9\left(3 - 4\cdot\frac{11}{36}\right) = 16.$$

Now AD is angle bisector than CD : BD = 9 : 15

$\therefore \quad BD = BC\cdot\left(\frac{BD}{BD+CD}\right)$

$$= 16\cdot\frac{15}{15+9} = \frac{16\times 15}{24} = 10$$

[Q.20] The figure in the complex plane given by $10z\bar{z}-3(z^2+z^{-2})+4i(z^2-z^{-2})=0$ is

[A] A straight line

[B] A circle

[C] A parabola

[D] An ellipse

[ANS] **A**

[SOL] Let $z=x+iy$ then $\bar{z}=x-iy$

Hence $z^2=x^2-y^2+2ixy$ and $\bar{z}^2=x^2-y^2-2ixy$

$\therefore \quad 10z\bar{z}-3(z^2+\bar{z}^2)+4i(z^2-\bar{z}2)=0$

$\Rightarrow \quad 10(x^2+y^2)-3.2(x^2-y^2)+4i\cdot(4ixy)=0$

$\Rightarrow \quad 10(x^2+y^2)-6x^2+6y^2-16xy)=0$

$\Rightarrow \quad 2x^2+8y^2-8xy=0$

$\Rightarrow \quad x^2-4xy+4y^2=0$

$\therefore \quad (x-2y)^2=0$

Which represents a straight line.

PART-I : PHYSICS

[Q.21] Students A, B and C measure the length of a room using 25 m long measuring tape of least count (LC) 0.5 cm, meter-scale of LC 0.1 cm and a foot-scale of LC 0.05 cm, respectively. If the specified length of the room is 9.5 m, then which of the following students will report the lowest relative error in the measured length?

[A] Student A

[B] Student B

[C] Student C

[D] Both, student B and C

[ANS] **A**

[SOL] Relative error $= \dfrac{\text{Deviation in measurment}}{\text{True measurement}}$

Error will be least in case of A.

[Q.22] Meena applies the front brakes while riding on her bicycle along a flat road. The force that slows her bicycle is provided by the

[A] Front tyre

[B] Road

[C] Rear tyre

[D] Brakes

[ANS] **B**

[SOL] External force is applied by the road only.

[Q.23] A proton and an antiproton come close to each other in vacuum such that the distance between them is 10 cm. Consider the potential energy to be zero at infinity. The velocity at this distance will be

[A] 1.17 m/s

[B] 2.3 m/s

[C] 3.0 m/s

[D] 23 m/s

[ANS] **A**

[SOL]

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$v = \left[\frac{e^2}{4\pi\varepsilon_0 mr}\right]^{1/2} = 1.17\ m/s$$

[Q.24] A point particle is acted upon by a restoring force $-kx^3$. The time period of oscillation is T when the amplitude is A. The time period for an amplitude 2A will be

[A] T

[B] T/2

[C] 2 T

[D] 4 T

[ANS] **B**

[SOL] $[F] = M^1L^1T^{-2} = kx^3$

$k = M^1L^{-2}T^{-2}$

$T \propto [M]^a[A]^b[k]^c$

a + c = 0

b – 2c = 0

– 2c = 1

$\Rightarrow \quad b = -1,\ c = -\frac{1}{2},\ a = \frac{1}{2}$

$T \propto \frac{1}{A}\sqrt{\frac{M}{k}}$

[Q.25] The output voltage (taken across the resistance) of aLCR series resonant circuit falls to half its peak value at a frequency of 200 Hz and again reaches the same value at 800 Hz. The bandwidth of this circuit is

[A] 200 Hz

[B] $200\sqrt{3}$ Hz

[C] 400 Hz

[D] 600 Hz

[ANS] **B**

[SOL] $\omega_1 L - \frac{1}{\omega_1 C} = \frac{1}{\omega_2 C} - \omega_2 L = \sqrt{3}R$

$(\omega_1 + \omega_2)L = \frac{(\omega_1 + \omega_2)}{\omega_1\omega_2 C}$

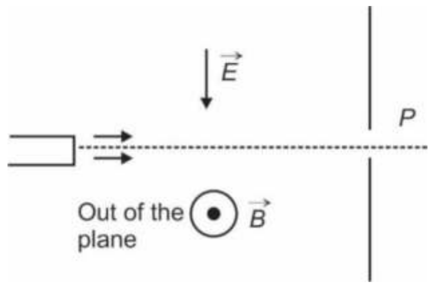
$\omega_1\omega_2 = \omega_0^2 \Rightarrow \omega_0 = \sqrt{\frac{1}{LC}} = 400\ Hz$

$$\frac{L}{R}=\frac{\sqrt{3}\,\omega_2}{\omega_0^2-\omega_2^2}=\frac{200\sqrt{3}}{400^2-200^2}$$

$$\frac{L}{R}=\frac{\sqrt{3}}{600}=\frac{1}{200\sqrt{3}}$$

Bandwidth of circuit $=\frac{R}{L}=200\sqrt{3}$

[Q.26] A collimated beam of charged and uncharged particles is directed towards a hole marked P on a screen as shown below. If the electric and magnetic fields as indicated below are turned on.



[A] Only particles with speed $\frac{E}{B}$ will go through the hole P.

[B] Only charged particles with speed $\frac{E}{B}$ and neutral particles will go through P.

[C] Only neutral particles will go through P.

[D] Only positively charged particles with speed $\frac{E}{B}$ and neutral particles will go through P.

[ANS] **C**

[SOL] $\vec{F}=q(\vec{E}+\vec{V}\times\vec{B})$

To go undeviated, $\vec{F}=0$

For the given direction of $\vec{V}$ none of the value of V can have $\vec{F}=0$

So, only neutral particle will go undeviated.

[Q.27] An engine runs between a reservoir at temperature 200 K and a hot body which is initially at temperature of 600 K. If the hot body cools down to a temperature of 400 K in the process, then the maximum amount of work that the engine can do (while working in a cycle) is (the heat capacity of the hot body is 1 J/K)

[A] 200(1 – In2) J

[B] 200(1 – In 3/2) J

[C] 200(1 + In3/2)J

[D] 200 J

[ANS] **B**

[SOL] $dQ = -CdT$

$$\eta = 1 - \frac{200}{T}$$

$$dW = \eta dQ = -CdT\left[1 - \frac{200}{T}\right]$$

$$W = \int dW = -1\int_{600}^{400} dT\left[1 - \frac{200}{T}\right]$$

$$W = 200 + 200\ln\left(\frac{2}{3}\right)$$

$$W = 200\left[1 - \ln\frac{3}{2}\right]$$

[Q.28] The clock tower ("ghantaghar") of Dehradun is famous for the sound of its bell, which can be heard, albeit faintly, upto the outskirts of the city 8 km away. Let the intensity of this faint sound be 30 dB. The clock is situated 80 m high. The intensity at the base of the tower is

[A] 60 dB

[B] 70 dB

[C] 80 dB

[D] 90 dB

[ANS] **B**

[SOL] $I \propto \frac{1}{r^2}$

$$\frac{I_1}{I_2} = \left[\frac{8000}{80}\right]^2 = 10000$$

Let intensity at base in decibel scale be x.

$$x - 30 = 10\log\frac{I_1}{I_2}$$

x = 70 dB

[Q.29] An initially uncharged capacitor C is being charged by a battery of emf E through a resistance R upto the instant, when the capacitor is charged to the potential $\frac{E}{2}$, the ratio of the work done by the battery to the heat dissipated by the resistor is given by,

[A] 2 : 1

[B] 3 : 1

[C] 4 : 3

[D] 4 : 1

[ANS] **C**

[SOL] Charge flown through battery $=\frac{CE}{2}$

Work done by battery $=\frac{CE^2}{2}$

Energy stored $=\frac{1}{2}CE^2-\frac{1}{2}\frac{CE^2}{4}$

$=\frac{3}{8}CE^2$

Required ratio $=\frac{4}{3}$

[Q.30] Consider a sphere of radius R with uniform charge density and total charge Q. The electrostatic potential distribution inside the sphere is given by $\theta_{(r)}=\frac{Q}{4\pi\varepsilon_0 R}(a+b(r/R)^C)$. Note that the zero of potential is at infinity. The values of (a, b, c) are

[A] $\left(\frac{1}{2},\frac{3}{2},1\right)$

[B] $\left(\frac{3}{2},-\frac{1}{2},2\right)$

[C] $\left(\frac{1}{2},-\frac{1}{2},1\right)$

[D] $\left(\frac{1}{2},-\frac{1}{2},2\right)$

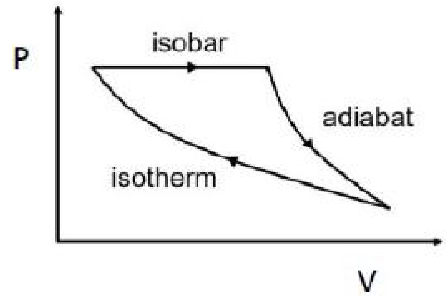
[ANS] **B**

[SOL] $V(r<R)=\frac{Q}{8\pi\varepsilon_0 R^3}(3R^2-r^2)$

$=\frac{Q}{4\pi\varepsilon_0 R}\left[\frac{3}{2}-\frac{r^2}{2R^2}\right]$

Comparing we get $a=\frac{3}{2}, c=2, b=\frac{-1}{2}$

[Q.31] The efficiency of the cycle shown below in the figure (consisting of one isobar, one adiabat and one isotherm) is 50%. The ratio, x, between the highest and lowest temperatures attained in this cycle obeys (the working substance is an ideal gas)



[A] $x = e^{x-1}$

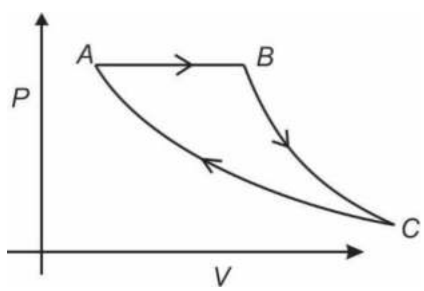
[B] $x^2 = e^{x-1}$

[C] $x = e^{x^2-1}$

[D] $x^2 = e^{x^2-1}$

[ANS] **B**

[SOL]



$T_B = xT_A = xT_C$

For BC, $T_B V_B^{\gamma-1} = T_C V_C^{\gamma-1}$

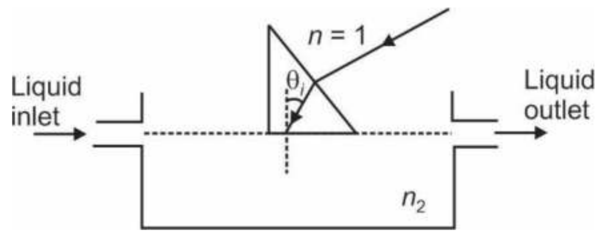
For AB, $\dfrac{V_B}{T_B} = \dfrac{V_A}{T_A}$

$\Rightarrow \quad V_B = xV_A \;\; \& \;\; V_C = x^{\left(\frac{\gamma}{\gamma-1}\right)} V_A$

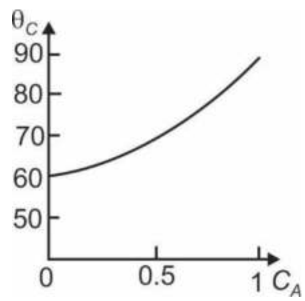
$$\eta = 1 - \frac{Q_{CA}}{Q_{AB}} = 1 - \frac{nRT_A \ln\left(\frac{V_C}{V_A}\right)}{\frac{n\gamma R}{\gamma-1}(T_B - T_A)} = 1 - \frac{\left(\frac{\gamma}{\gamma-1}\right)\ln x}{\left(\frac{\gamma}{\gamma-1}\right)(x-1)}$$

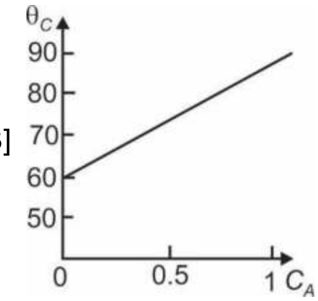
$$\eta = \frac{1}{2} \Rightarrow \frac{\ln x}{x-1} = \frac{1}{2} \Rightarrow x^2 = e^{x-1}$$

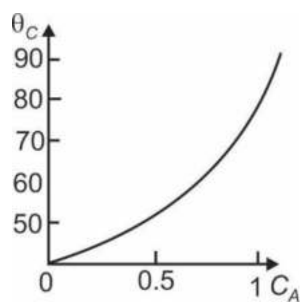
[Q.32] A right-angled isosceles prism is held on the surface of a liquid composed of miscible solvents A and B of refractive index n_A = 1.50 and n_B = 1.30, respectively. The refractive index of prism is n_p = 1.5 and that of the liquid is given by $n_L = C_A n_A + (1 - C_A)n_B$, where C_A is the percentage of solvent A in the liquid.

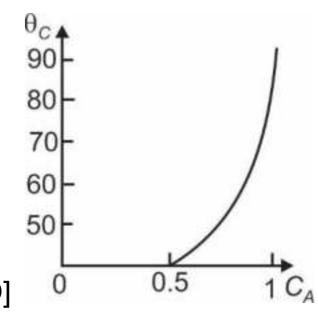


If θ_C is the critical angle at prism-liquid interface, the plot which best represents the variation of the critical angle with the percentage of solvent is:

[A]


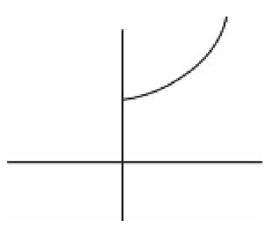
[B]


[C]


[D]


[ANS] **A**

[SOL]



$$n_L = C_A(1.5) + (1 - C_A) \times 1.3$$

$$n_L = 1.3 + 0.2 \times C_A$$

$$\sin\theta_C = \frac{n_L}{1.5} = \frac{1.3 + 0.2C_A}{1.5}$$

$\Rightarrow$ θ_C is increasing function

[Q.33] Instead of angular momentum quantization a student posits that energy is quantized as $E = \frac{-E_0}{n}(E_0 > 0)$ and n is a positive integer. Which of the following options is correct?

[A] The radius of the electron orbit is $r \propto \sqrt{n}$

[B] The speed of the electron is $v \propto \sqrt{n}$

[C] The angular speed of the electron is $\omega \propto \frac{1}{n}$

[D] The angular momentum of the electron is $\propto \sqrt{n}$

[ANS] **D**

[SOL] $E = -\frac{E_0}{n}$

$$\frac{mv^2}{r_n} = \frac{(k)Ze^2}{r_n^2} \quad ....(1)$$

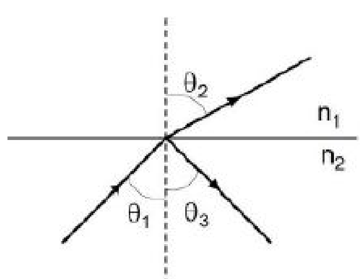
$$\frac{1}{2}mv^2 - \frac{kZe^2}{r_n} = -\frac{E_0}{n} \quad ....(2)$$

From (1), $\frac{1}{2}mv^2 = \frac{1}{2}\frac{kZe^2}{r_n}$ .....(3)

From (2), $-\frac{kZe^2}{2r_n} = -\frac{E_0}{n}$

$\Rightarrow$ $L = mv_n r_n = \frac{c}{\sqrt{n}}$

[Q.34] A monochromatic beam of light is incident at the interface of two materials of refractive index n_1 and n_2 as shown. If $n_1 > n_2$ and θ_C is the critical angle then which of the following statements is NOT true?



[A] $\theta_1 = \theta_3$ for all values of θ_1

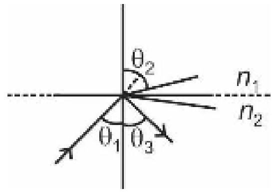
[B] $\cos\theta_2$ is imaginary for $\theta_1 > \theta_C$

[C] $\cos\theta_2 = 0$ for $\theta_1 = \theta_C$.

[D] $\cos\theta_3$ is imaginary for $\theta_1 = \theta_C$

[ANS] **D**

[SOL] Based on the diagram given, n_1 should be less than n_2



TIR will occur at $\theta_1 > \theta_C$. There will be no refracted ray.

At $\theta_1 = \theta_C$, $\theta_2 = 90^o$ hence $\cos\theta_2 = 0$

$\theta_1 > \theta_C$ no refracted ray, hence $\cos\theta_2$ is imaginary.

Based on law of reflection angle of incidence is equal to angle of reflection.

All options are true except option (D).

[Q.35] The intensity of light from a continuously emitting laser source operating at 638 nm wavelength is modulated at 1 GHz. The modulation is done by momentarily cutting the intensity off with a frequency of 1 GHz. What is the farthest distance apart two detectors can be placed in the line of the laser light, so that they can see the portions of the same pulse simultaneously? (Consider the speed of light in air 3×10^8 m/s)

[A] 30 μm

[B] 30 cm

[C] 3 m

[D] 30 m

[ANS] **B**

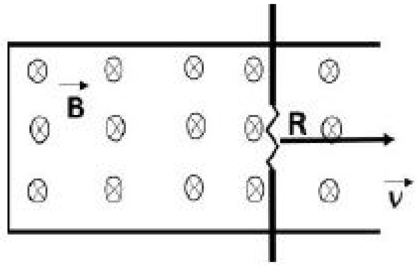
[SOL] The farthest distance apart two detectors should be of order of wavelength of light

So, $d \approx \lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^9}$

$= 0.3$ m

≈ 30 cm

[Q.36] A conducting rod, with a resistor of resistance R, is pulled with constant speed v on a smooth conducting rail as shown in figure. A constant magnetic field $\vec{B}$ is directed into the page. If the speed of the bar is doubled, by what factor does the rate of heat dissipation across the resistance R change?



[A] 0

[B] $\sqrt{2}$

[C] 2

[D] 4

[ANS] **D**

[SOL] $P = \frac{E^2}{R} = \frac{(B\ell v)^2}{R}$

$P_1 = CV^2$

$P_2 = C(2V)^2 = 4P_1$

[Q.37] The time period of a body undergoing simple harmonic motion is given by $T = p^a D^b S^c$, where p is the pressure, D is density and S is surface tension. The values of a, b and c respectively are

[A] $1, \frac{1}{2}, \frac{3}{2}$

[B] $\frac{3}{2}, -\frac{1}{2}, 1$

[C] $1, -\frac{1}{2}, \frac{3}{2}$

[D] $-\frac{3}{2}, \frac{1}{2}, 1$

[ANS] **D**

[SOL] [T] = [T]

[P] = $[ML^{-1}T^{-2}]$

[D] = $[ML^{-3}]$

[S] = $\left[\frac{MLT^{-2}}{L}\right] = [MT^{-2}]$

$P^a D^b S^c = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [MT^{-2}T]^C$

$\Rightarrow \quad [T] = [M^{a+b+c} L^{-a-3b} T^{-2a-2c}]$

Compare both sides we get

$a = -\frac{3}{2}, b = \frac{1}{2}, c = 1$

[Q.38] Consider the following statements regarding the real images formed with a converging lens.

I. Real images can be seen only if the image is projected onto the screen

II. The real image can be seen only from the same side of the lens as that on which the object is positioned.

III. Real images produced by converging lenses are not only laterally but also longitudinally inverted as with mirrors.

Which of the above statement/statements is/are incorrect?

[A] Only I and III

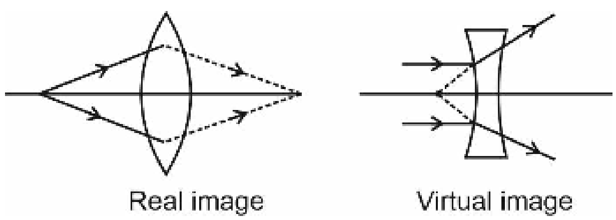
[B] All three

[C] None

[D] Only II

[ANS] **B**

[SOL] For real image, point of intersection of refracted rays must be real



[Q.39] A zinc ball of radius, R = 1 cm charged to a potential –0.5 V. The ball is illuminated by a monochromatic ultraviolet (UV) light with a wavelength 290 nm. The photoelectric threshold for zinc is 332 nm. The potential of ball after a prolonged exposure to the UV is

[A] – 0.5 V

[B] 0 V

[C] 0.54 V

[D] 0.79 V

[ANS] **C**

[SOL] After long time interval emission will stop, when potential acquired is equal to stopping potential.

$$W \text{ (work function)} = \frac{hc}{\lambda_{th}} = \frac{1242}{332} \text{eV}$$

$$= 3.74 \text{ eV}$$

$$E_{Ph} = \frac{hc}{\lambda_{Ph}} = \frac{1242}{290} = 4.28 \text{ eV}$$

$K_{max} = E_{Ph} - W$

$= 0.54$ eV

$\Rightarrow$ V = 0.54 Volt

[Q.40] A source simultaneously emitting light at two wavelengths 400 nm and 800 nm is used in the Young's double slit experiment. If the intensity of light at the slit for each wavelength is I_0, then the maximum intensity that can be observed at any point on the screen is

[A] I_0

[B] $2I_0$

[C] $4I_0$

[D] $8I_0$

[ANS] **D**

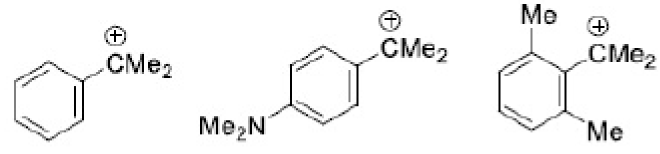
[SOL] Maximum intensity of light due to single wavelength is

$$I_1^{max} = \left(\sqrt{I_0} + \sqrt{I_0}\right)^2 = 4I_0$$

So maximum intensity due to single light in $4I_0$, so maximum possible intensity will be $4I_0 + 4I_0 = 8I_0$

PART-I : CHEMISTRY

[Q.41] The stability of



I II III

Follows the order :

[A] I > II > III

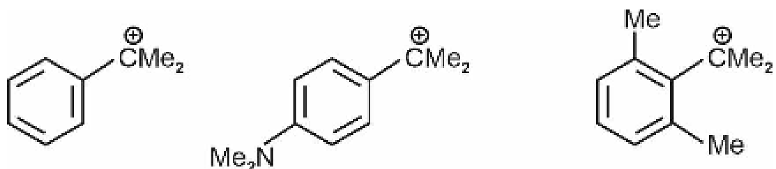
[B] II > I > III

[C] II > III > I

[D] III > II > I

[ANS] **B**

[SOL]



I II III

Structure-II has +M effect of $-NMe_2$ group, hence maximum stability among the given set of cations. In structure-III due to SIR effect $-\overset{\oplus}{C}Me_2$ moves out of plane of benzene ring hence stability decreases due to loss of conjugation.

Finally correct order should be : II > I > III

[Q.42] Among the following, the biodegradable polymer is :

[A] Polylactic acid

[B] Polyvinyl chloride

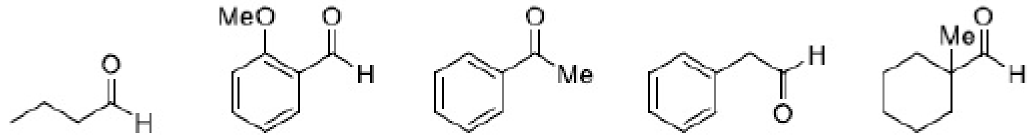
[C] Bakelite

[D] Teflon

[ANS] **A**

[SOL] Fact based question. Answer is Polylactic acid.

[Q.43] Among the following,



I II III IV V

the compounds which can be reduced with formaldehyde and conc. Aq. KOH, are :

[A] Only II and V

[B] Only I and V

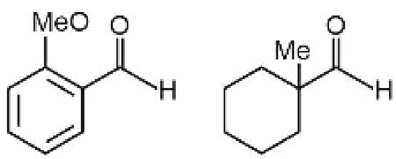
[C] Only II and III

[D] Only I, II and IV

[ANS] **A**

[SOL] In crossed Cannizzaro reaction, more reactive aldehyde undergoes oxidation and less reactive will undergo reduction. In Cannizzaro reaction no $\alpha - H$ should be present.

So the following compounds can be reduced by formaldehyde and conc. aq KOH



II V

[Q.44] An organic compound that is commonly used for sanitizing surfaces is :

[A] Acetylsalicylic acid

[B] Chloramphenicol

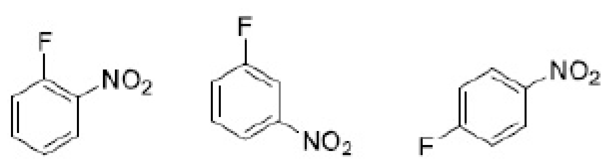
[C] Aspartame

[D] Cetyltrimethyl ammonium bromide

[ANS] **D**

[SOL] Fact based. Cetyltrimethyl ammonium bromide is a popular cationic detergent.

[Q.45] The rates of reaction of NaOH with



I II III

Follow the order :

[A] II > I > III

[B] II > III > I

[C] I > III > II

[D] III > II > I

[ANS] C

[SOL] Aryl halides do not undergo nucleophilic substitution reaction under normal conditions. But when –M/–I groups present at ortho or para position can enhance reactivity due to stabilization of intermediate.

So compound I and III are reactive with NaOH compared to II. Final order should be I > III > II.

[Q.46] The most suitable reagent for the conversion of 2-phenylpropanamide into 1-phenylethylamine is :

[A] H_2, Pd/C

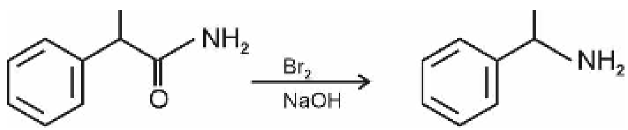
[B] Br_2, NaOH

[C] $LiAlH_4$, Et_2O

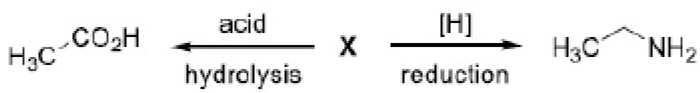
[D] $NaBH_4$, MeOH

[ANS] B

[SOL] Hoffmann bromamide degradation reaction.



[Q.47] The compound X in the following reaction scheme



[A] Acetonitrile

[B] Methyl isocyanide

[C] Acetaldehyde

[D] Nitromethane

[ANS] A

[SOL] Acetonitrile is $CH_3 - C \equiv N$

$$CH_3 - C \equiv N \xrightarrow{H_3^+O} CH_3 - COOH + NH_4^+$$

$$CH_3 - C \equiv N = \frac{\text{Reduction}}{[H]} CH_3 - CH_2 - NH_2$$

[Q.48] A nucleus X captures a β particle and then emits a neutron and γ ray to form Y. X and Y are :

[A] Isomorphs

[B] Isotopes

[C] Isobars

[D] Isotones

[ANS] **D**

[SOL] ${}_{z}X^{A} + {}_{-1}e^{0} \longrightarrow {}_{z-1}X^{A} \xrightarrow{-{}_{0}N^{1}} {}_{z-1}X^{A-1} \xrightarrow{\gamma\text{ emission}} {}_{z-1}X^{A-1}(Y)$

A = Mass number of X

Z = Atomic number of X

Number of neutrons in X = A – Z

Number of neutrons in Y = A – 1 – (Z – 1) = A – Z

i.e., X & Y are isotones (have same number of neutrons)

[Q.49] The boiling point (in °C) of 0.1 molal aqueous solution of $CuSO_4.5H_2O$ at 1 bar is closest to

[Given : Ebullioscopic (molal boiling point elevation) constant of water, K_b = 0.512 K Kg mol^{-1}]

[A] 100.36

[B] 99.64

[C] 100.10

[D] 99.90

[ANS] **C**

[SOL] 0.1 molal $CuSO_4.5H_2O$ aqueous solution.

$\Rightarrow \quad \Delta T_b = 2 \times 0.512 \times 0.1$

$\Rightarrow \quad T_b - 100^{\circ}C = 0.1024$

$\Rightarrow \quad T_b - 100.1$

[Q.50] A weak acid is titrated with a weak base. Consider the following statements regarding the pH of the solution at the equivalence point

(i) pH depends on the concentration of acid and base

(ii) pH is independent of the concentration of acid and base

(iii) pH depends on the pK_a of acid and pK_b of base

(iv) pH is independent of the pK_a of acid and pK_b of base

The correct statements are

[A] Only (i) and (iii)

[B] Only (i) and (iv)

[C] Only (ii) and (iii)

[D] Only (ii) and (iv)

[ANS] **C**

[SOL] pH at equivalence point for the titration of weak acid with weak base is given by the relation :

$$pH = \frac{1}{2}(pK_w + pK_a - pK_b)$$

i.e., pH is independent of concentration of acid and base. pH depends on the pK_a of acid and pK_b of base.

$\therefore$ Correct statements : (ii) & (iii)

[Q.51] Products are favoured in a chemical reaction taking place at a constant temperature and pressure. Consider the following statements:

(i) The change in Gibbs energy for the reaction is negative

(ii) The total change in Gibbs energy for the reaction and the surroundings is negative

(iii) The change in entropy for the reaction is positive.

(iv) The total change in entropy for the reaction and the surroundings is positive.

The statements which are ALWAYS true are:

[A] Only (i) and (iii)

[B] Only (i) and (iv)

[C] Only (ii) and (iv)

[D] Only (ii) and (iii)

[ANS] **B**

[SOL] Reaction is taking place at constant temperature and pressure and products formation is favoured.

i.e., reaction is spontaneous.

$\therefore \quad \Delta S_T = \Delta S_{sys} + \Delta S_{surr} > 0$

At constant T & P

$\Delta G_{reaction} < 0$

Correct statements : (i) & (iv)

[Q.52] A mixture of toluene and benzene forms a nearly ideal solution. Assume P_B^o and P_T^o to be the vapor pressures of pure benzene and toluene, respectively. The slope of the line obtained by plotting the total vapor pressure to the mole fraction of benzene is

[A] $P_B^o - P_T^o$

[B] $P_T^o - P_B^o$

[C] $P_B^o - P_T^o$

[D] $\left(P_B^o - P_T^o\right)/2$

[ANS] **A**

[SOL] According to Raoult's law

$P_{Total} = \chi_B.P_B^o + \chi_T.P_T^o$

$\chi_B + \chi_T = 1$

$\therefore \quad P_{Total} = \chi_B P_B^o + (1-\chi_B)P_T^o$

$= P_T^o + \left(P_B^o - P_T^o\right)\chi_B$

$\therefore$ The graph of P_{Total} vs χ_B will be straight line with slope equal to $\left(P_B^o - P_T^o\right)$

[Q.53] Upon dipping a copper rod, the aqueous solution of the salt that can turn blue is:

[A] $Ca(NO_3)_2$

[B] $Mg(NO_3)_2$

[C] $Zn(NO_3)_2$

[D] $AgNO_3$

[ANS] D

[SOL] $Cu(s) \longrightarrow Cu^{2+}(aq) + 2e^-; E^o = -0.34$ V

$Ag^+(aq) + e^- \longrightarrow Ag(s) \ ; E^o = 0.80$ V

$\therefore$ Only $AgNO_3$ among the given will be able to oxidise Cu to Cu^{2+} (responsible for blue colour).

[Q.54] Treatment of alkaline $KMnO_4$ solution with KI solution oxidizes iodide to :

[A] I_2

[B] IO_4^-

[C] IO_3^-

[D] IO_2^-

[ANS] C

[SOL] $2MnO_4^- + I^- + H_2O \longrightarrow IO_3^- + 2MnO_2 + 2OH^-$

[Q.55] If an extra electron is added to the hypothetical molecule C_2, this extra electron will occupy the molecular orbital.

[A] π_{2p} *

[B] π_{2p}

[C] σ_{2p} *

[D] σ_{2p}

[ANS] D

[SOL] C_2 configuration

$\sigma_{1s}^2 < \sigma_{1s}^{*2} < \sigma_{2s}^2 < \sigma_{2s}^{*2} < \left(\pi 2p_y^2 = \pi 2p_x^2\right) < \sigma 2p_z$

The next electron will come in $\sigma 2p_z$ orbital.

[Q.56] Among the following the square planar geometry is exhibited by :

[A] $CdCl_4^{2-}$

[B] $Zn(CN)_4^{2-}$

[C] $PdCl_4^{2-}$

[D] $Cu(CN)_4^{3-}$

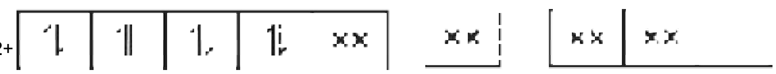
[ANS] **C**

[SOL] $CdCl_4^{2-}$ $\Rightarrow$ $Cd^{2+} : 3d^{10}$

$[Zn(CN)_4]^{2-}$ $\Rightarrow$ $Zn^{2+} : 3d^{10}$

$[Cu(CN)_4]^{3-}$ $\Rightarrow$ $Cu^{+} : 3d^{10}$

$[PdCl_4]^{2-}$ $\Rightarrow$ $pd^{2+} : 4d^{8}$ (element is of 4d series)

Pd^{2+} 

$Dsp^2 \Rightarrow$ Square planar

[Q.57] The correct pair of orbitals involved in π-bonding between metal and CO in metal carbonyl complexes is:

[A] Metal d_{xy} and carbonyl π_x *

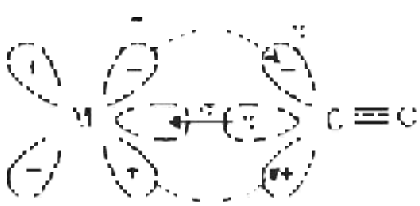
[B] Metal d_{xy} and carbonyl π_x

[C] Metal $d_{x^2-y^2}$ and carbonyl π_x *

[D] Metal $d_{x^2-y^2}$ and carbonyl π_x

[ANS] **A**

[SOL]



d_{xy} orbitals of metal (M)

Orbitals involved in π-bonding between metal and CO are d_{xy} of metal and π^* of CO.

[Q.58] The magnetic moment (in μ_B) of $[Ni(dimethylglyxoimate)_2]$ complex is closest to:

[A] 5.37

[B] 0.00

[C] 1.73

[D] 2.25

[ANS] **B**

[SOL] [Ni (dimethylglyxoimate)$_2$] $\Rightarrow$ [Ni(DMG)$_2$]

Ni^{2+} with strong field ligand (DMG) form low spin complex.

$\Rightarrow$ No. of unpaired electron = 0

$\mu_B = \sqrt{n(n+2)}$ B.M. $= 0$

[Q.59] A compound is formed by two elements M and N. Element N forms hexagonal closed pack array with 2/3 of the octahedral holes occupied by M. The formula of the compound is :

[A] M_4N_3

[B] M_2N_3

[C] M_3N_2

[D] M_3N_4

[ANS] **B**

[SOL] Element N forms HCP

$\therefore$ No. of N-atoms (or ions) per unit cell $= \frac{1}{2}\times 2 + \frac{1}{6}\times 12 + 1\times 3$

= 6

$\therefore$ No. of octahedral voids per unit cell = 6

No. of M-atoms (or ions) per unit cell $= \frac{2}{3}\times 6 = 4$

$\therefore$ Formula of compound : M_4N_6 or M_2N_3

[Q.60] If the velocity of the revolving electron of He^+ in the first orbit (n = 1) is v, the velocity of the electron in the second orbit is

[A] v

[B] 0.5 v

[C] 2v

[D] 0.25 v

[ANS] **B**

[SOL] Velocity of electron in a unielectronic species as per Bohr's model is

$$V = V_0 \times \frac{Z}{n} (v_0 = 2.18\times 10^6\ m/s = \text{constant})$$

For He^+ and n = 1

$$v = v_0 \times \frac{2}{1}$$

$$\Rightarrow \quad v_0 = \frac{v}{2}$$

For He^+ & n = 2

$$v' = V_0 \times \frac{2}{2} = v_0 = \frac{v}{2}$$

$$v' = 0.5\ v$$

PART-I : BIOLOGY

[Q.61] Species with high fecundity, high growth rates, and small body sizes are typically

[A] Endangered species

[B] Keystone species

[C] K-selected species

[D] r-selected species

[ANS] **D**

[SOL] Environmental instability or unpredictability favours quick reproduction and renders useless competitive adaptations in r-selected species.

Among the traits that are thought to characterize r-selection are high fecundity, high growth rates and small body sizes.

[Q.62] When RNase enzyme is denatured by adding urea, which ONE of the following combinations of bonds would be disrupted?

[A] Ionic and disulphide bonds

[B] Ionic and hydrogen bonds

[C] Hydrogen and peptide bonds

[D] Peptide and disulphide bonds

[ANS] **B**

[SOL] Treatment of RNase with urea disrupts hydrogen bonds and ionic bonds and results in denaturation of the protein, so the correct answer is option (B). Option (A), (C) and (D) are incorrect as covalent bonds such as peptide bonds and disulphide bonds are broken upon degradation of protein when the primary structure is destroyed.

[Q.63] The function of aposematic colouration is to

[A] Attract mates

[B] Camouflage

[C] Scare off competitors

[D] Warn predators

[ANS] **D**

[SOL] Aposematic coloration is a form of coloration which discourages a predator from eating an organism (its prey).

So predators are warned by this method.

There is often a sting, poison or painful bite associated to it.

[Q.64] Maize and rice genomes have diploid chromosome number of 20 and 24, respectively. In the absence of crossing over and mutations, which ONE of the following is CORRECT about the genetic variation among their offspring?

[A] Maize < rice

[B] Maize = rice > 0

[C] Maize = rice = 0

[D] Maize > rice

[ANS] **A**

[SOL] When there is no crossing over and mutations then the only left option for introducing variations among offsprings is random orientation/arrangement of homologous chromosomes on metaphasic plate. This event completely random and leads to independent assortment of homologous chromosomes.

The number of variations is dependent on the number of chromosomes making up a set. Therefore the number of possible alignments is 2^n in a diploid cell where n is number of chromosomes per haploid set.

So In maize = $2^n = 2^{10}$ arrangements and in Rice = $2^n : 2^{12}$ arrangements.

Hence Maize < Rice

[Q.65] The exponent z of the species-area curve measured at continental scales is

[A] Smaller than the value of z at regional scales

[B] Equal to the value of z at regional scales

[C] Greater than the value of z at regional scales

[D] Unrelated to the value of z at regional scales

[ANS] **C**

[SOL] Alexander von Humboldt observed that within a region species richness increases with increasing explored area but only upto a certain limit. If we analyse the species area relationship among very large areas like continents, then slope of the line (Z) will be much steeper means greater Z value than at regional scales.

Hence the correct option is (C)

[Q.66] The pH of an aqueous solution of 10^{-8} M HCl is

[A] 6.0

[B] Between 6.9 – 7.0

[C] Between 7.0 – 7.1

[D] 8.0

[ANS] **B**

[SOL]

10^{-8}M HCl $\Rightarrow$ $HCl(aq) \rightarrow H^{+}(aq) + Cl^{-}(aq)$

10^{-8}	0	0
–	10^{-8}	10^{-8}

$\Rightarrow$ $H_2O(l) \rightleftharpoons H^{+}(aq) + OH^{-}(aq)$

$(x + 10^{-8})$ x

$K_w = [H^+][OH^-] = (x + 10^{-8})x = 10^{-14}$

On solving, $x = 9.5 \times 10^{-8}$

So, $[H^+]_{Total} = x + 10^{-8} = 9.5 \times 10^{-8} + 10^{-8} = 1.05 \times 10^{-7}$

$pH = -\log(H^+) = -\log(1.05 \times 10^{-7}) = 6.98$

[Q.67] Which ONE of the following can NOT cause eutrophication of lakes?

[A] Introduction of invasive floating plants

[B] Discharge of fertilizer-rich agricultural waste

[C] Natural ageing of lakes

[D] Discharge of industrial waste

[ANS] **A**

[SOL] Eutrophication is aging of lake due to nutrient enrichment particularly with nitrogen and phosphorus.

Introduction of invasive floating plants is not the cause of eutrophication.

Though these plants will grow well on eutrophic lakes.

[Q.68] Which ONE of the following polymerases transcribes 5S rRNA?

[A] RNA Pol I

[B] RNA Pol III

[C] RNA Pol II

[D] RNA Pol IV

[ANS] **B**

[SOL] RNA polymerase I transcribes all types of rRNA except 5S rRNA. It is transcribed by RNA pol III which also transcribes tRNA.

[Q.69] Which ONE of the following statements about rennin is CORRECT ?

[A] It is secreted by adrenal glands.

[B] It converts angiotensinogen to angiotensin.

[C] It is secreted by peptic cells of gastric glands into the stomach

[D] It is a hormone

[ANS] **C**

[SOL] Rennin is a proteolytic enzyme secreted by peptic cells of gastric glands. It helps in the digestion of milk proteins in infants.

[Q.70] When one goes from a brightly lit area to a dimly lit room our eyes adjusts slowly, thereby regaining the clarity of vision. Which ONE of the following explains this process?

[A] Regeneration of rhodopsin in the rod cells

[B] Bleaching of rhodopsin

[C] Constriction of the pupil

[D] Increase in the number of rod cells

[ANS] **A**

[SOL] When one goes from a brightly lit area to a dimly lit room, Rhodopsin regeneration takes place. This process is called dark adaptation, so correct answer is option (A).

Option (B) is incorrect as bleaching of rhodopsin in the rod cells takes place in brightly lit area.

[Q.71] In a diploid population at Hardy-Weinberg equilibrium, consider a locus with two alleles. The frequencies of these two alleles are denotes by p and q, respectively. Heterozygosity in this population is maximum at.

[A] p = 0.25, q = 0.75

[B] p = 0.4, q = 0.6

[C] p = 0.6, q = 0.4

[D] p = 0.5, q = 0.5

[ANS] **D**

[SOL] According to Hardy -Weinberg principle : $p^2 + q^2 + 2pq = 1$

where 2pq represents heterozygotes.

Options	p	q	2pq
A	0.25	0.75	0.375
B	0.4	0.6	0.48
C	0.6	0.4	0.48
E	0.5	0.5	0.50

Thus, as heterozygosity in this population is maximum at p = 0.5, and q = 0.5, the correct answer is option (D)

[Q.72] An enzyme with optimal activity at pH 2.0 and 37°C is most likely to be

[A] Lysozyme from hen egg white

[B] Trypsin from cattle

[C] DNA polymerase from *Thermus aquaticus*

[D] Pepsin from humans

[ANS] **D**

[SOL] The optimal stable pH for lysozyme is 7.5 so option (A) is incorrect.

Optimum pH for trypsin is ~ 7.8, so option (B) is incorrect.

DNA polymerase from *Thermus aquaticus* is called Taq DNA polymerase. It is a heat stable enzyme that works best at high temperature, so option (C) is incorrect.

The correct answer is option (D) as pepsin secreted, from peptic cells of gastric glands in stomach works effectively at pH ~ 1.8.

[Q.73] While adjusting to varying environmental temperature, plants incorporate in their plasma membrane

[A] More saturated fatty acids in cold and more unsaturated fatty acids in hot environment.

[B] More unsaturated fatty acids in cold and more saturated fatty acids in hot environment.

[C] More saturated fatty acids in both cold and hot environment.

[D] More unsaturated fatty acids in both cold and hot environment.

[ANS] **B**

[SOL] The fluidity of plasma membrane should be maintained in cold and hot environment.

A cold environment tends to compress membranes composed largely of saturated fatty acids making them less fluid. So in cold environment, the proportion of unsaturated fatty acids should be larger as kinks in the tail push adjacent phospholipid molecules away and maintains fluidity in membrane. Saturated fatty acid makes the membrane dense and fairly rigid.

So the correct option is (B)

[Q.74] Which ONE of the following terms in NOT used while describing human vertebra?

[A] Lumber

[B] Sacral

[C] Thoracic

[D] Tarsal

[ANS] **D**

[SOL] Human vertebral column is differentiated into cervical, thoracic, lumbar, sacral and coccygeal.

So, option (A), (B) and (C) cannot be the correct answer.

The correct answer is option (D) as Tarsal (ankle) is a bone of hindlimb.

[Q.75] Assume a population that has reached herd immunity for an infectious disease. If an infected individual is introduced to this population, which of the following is most likely to occur?

[A] The infection will spread exponentially across the population.

[B] The infection will spread linearly across the population.

[C] A few individuals may get infected, but the infection will not spread across the population.

[D] No other individual will be infected by the disease.

[ANS] **C**

[SOL] Herd immunity is a form of indirect protection from infectious diseases. When a sufficient percentage of a population has become immune to an infection, it breaks the chain of transmission thereby reducing the likelihood of infection for individuals who lack immunity against that infection. So, the correct answer is option (C).

[Q.76] Match the type of cells in **Column I** with the organs they are part of, listed in **Column II**.

	Column I		**Column II**
P.	Chondroblast	i.	Bone
Q.	Osteoclast	ii.	Brain
R.	Microglia	iii.	Cartilage
S.	Pneumocyte	iv.	Lung

Choose the CORRECT combination.

[A] P-iii, Q-i, R-ii, S-iv

[B] P-ii, Q-i, R-iii, S-iv

[C] P-iv, Q-iii, R-ii, S-i

[D] P-iii, Q-ii, R-iv, S-i

[ANS] **A**

[SOL] Chondroblasts are cartilage forming cells whereas osteoclasts are macrophages of bones. Microglia are macrophages of neural tissue. Pneumocytes are present in lung alveoli so the correct answer is option (A)

[Q.77] A bacterial culture was started with an inoculum of 10 cells. What will be the number of cells at the end of 10 cycles of division, assuming that every progeny cell undergoes division in each cycle ?

[A] 100

[B] 1024

[C] 2048

[D] 10240

[ANS] **D**

[SOL] Since, a bacterial culture was started with an inoculum of 10 cells (given)

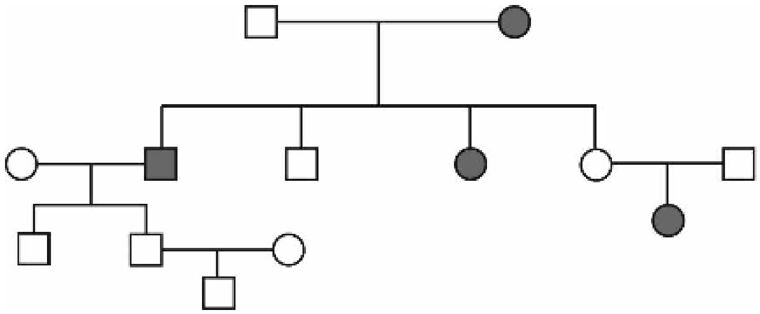
Number of cells at the end of 10 cycles of division

$= 2^{10} \times 10$ (One parent cell divides into two daughter cells)

$= 1024 \times 10$

$= 10240$

[Q.78] The following family tree traces the occurrence of a rare genetic disease. The filled symbols signify the individuals with the disease, whereas the open symbols signify healthy individuals.



Based on this information, the disease is most likely to be

[A] Autosomal, dominant

[B] Autosomal, recessive

[C] X-linked, recessive

[D] X-linked, dominant

[ANS] **B**

[SOL] Given pedigree shows autosomal recessive disorder. It cannot be dominant as not seen in every generation. It cannot be x-linked recessive as mother is affected in I generation but its one of the sons is normal which is not possible.

So the correct option is (B).

[Q.79] Which ONE of the following statement is CORRECT about the mechanism of action of penicillin?

[A] It inhibits transcription

[B] It hydrolyses cell wall

[C] It inhibits cell wall biosynthesis

[D] It inhibits translation

[ANS] **C**

[SOL] Penicillin is the first antibiotic discovered. It is effective against cell wall containing monerans as it inhibits cell wall biosynthesis. Hence the smallest organism, mycoplasma is insensitive to penicillin as it is wall-less.

[Q.80] Leaf extract from an infected plant was passed through a filter with a pore size of 0.05 μm diameter. The infectious agent was detected in the filtrate. Which ONE of the following is the likely infectious agent ?

[A] Bacteria

[B] Virus

[C] Nematode

[D] Fungus

[ANS] **B**

[SOL] Generally the bacterial filter is with a pore size of 0.05 μm diameter. It is given that infectious agent was detected in the filtrate. So it should be smaller than bacteria, that‟s why it has come out in the filtrate.

Hence the correct option is (B) i.e., virus

PART-II : MATHEMATICS

[Q.81] Let $a = \sum_{n=101}^{200} 2^n \sum_{k=101}^{n} \frac{1}{k!}$ and $b = \sum_{n=101}^{200} \frac{2^{201} - 2^n}{n!}$.

Then $\frac{a}{b}$ is

[A] 1

[B] $\frac{3}{2}$

[C] 2

[D] $\frac{5}{2}$

[ANS] **A**

[SOL] $a = 2^{101}\left(\frac{1}{\underline{|101}}\right) + 2^{102}\left(\frac{1}{\underline{|101}} + \frac{1}{\underline{|102}}\right) + 2^{103}\left(\frac{1}{\underline{|101}} + \frac{1}{\underline{|102}} + \frac{1}{\underline{|103}}\right) + \ldots + 2^{200}\left(\frac{1}{\underline{|101}} + \frac{1}{\underline{|102}} + \ldots + \frac{1}{\underline{|200}}\right)$

$\Rightarrow \quad a = \frac{1}{\underline{|101}}\left(2^{101} + 2^{102} + \ldots + 2^{200}\right) + \frac{1}{\underline{|102}}\left(2^{102} + 2^{103} + \ldots + 2^{200}\right) + \ldots + \frac{1}{\underline{|200}}\left(2^{200}\right)$

$\Rightarrow \quad a = \sum_{n=101}^{200} \frac{1}{\underline{|n}}\left(2^n + 2^{n+1} + \ldots + 2^{200}\right)$

$\Rightarrow \quad a = \sum_{n=101}^{200} \frac{1}{\underline{|n}}\left(\frac{2^n(2^{201-n} - 1)}{2-1}\right) = \sum_{n=101}^{200} \frac{2^{201} - 2^n}{\underline{|n}}$

$\Rightarrow \quad a = b$

[Q.82] Let a, b, c be non-zero real roots of the equation $x^3 + ax^2 + bx + c = 0$. Then

[A] There are infinitely many such triples *a, b, c*

[B] There is exactly one such triple *a, b, c*

[C] There are exactly two such triples *a, b, c*

[D] There are exactly three such triples *a, b, c*

[ANS] **C**

[SOL] $\because$ abc = – c …. (i)

ab + bc + ca = b …..(ii)

and a + b+ c = – a …..(iii)

from (i), ab = –1 as $c \neq 0$

So from (i), $c = -2a + \frac{1}{a}$

and from (ii) we get, $-1+2-\frac{1}{a^2}-2a^2+1=-\frac{1}{a}$

$\Rightarrow$ $2a^4 - 2a^2 - a + 1 = 0$

$\Rightarrow$ $(a - 1)(2a^3 + 2a^2 - 1) = 0$

From here we get only two real and non-zero values of a, hence there exists two triplets of (a, b, c).

[Q.83] Let $f(x) = \sin x + (x^2 - 3x^2 + 4x - 2)\cos x$ for $x \in (0, 1)$. Consider the following statements

I. f has a zero in (0, 1)

II. f is monotone in (0, 1)

Then

[A] I and II are true

[B] I is true and II is false

[C] I is false and II is true

[D] I and II are false

[ANS] **A**

[SOL] $\because$ $f(x) = (x-1)^3\cos x + (x-1)\cos x + \sin x$

$\Rightarrow$ $f'(x) = 3(x-1)^2\cos x + 2\cos x - \sin x[(x-1)^3 + (x-1)]$

$\Rightarrow$ $f'(x) = \cos x\underbrace{[3(x-1)^2+2]}_{\text{positive}} + \underbrace{(1-x)}_{\text{positive in }(0,1)}\sin x\underbrace{[(x-1)^2+1]}_{\text{positive}}$

$\because$ $\sin x > 0$ and $\cos x > 0$ $\forall$ $x \in (0, 1)$

So, $f'(x) > 0 \Rightarrow f(x)$ monotonically increasing.

Also f(0) = – 2 and f(1) = sin 1 > 0

Hence f(x) has exactly one root in (0, 1)

[Q.84] Let A be a set consisting of 10 elements. The number of non-empty relations from A to A that are reflexive but not symmetric is

[A] $2^{89} - 1$

[B] $2^{89} - 2^{45}$

[C] $2^{45} - 1$

[D] $2^{90} - 2^{45}$

[ANS] **D**

[SOL] $\because$ A × A contains 100 ordered pairs (a, b) out of which 10 ordered pairs are such that a = b.

For a relation to be reflexive (a, a) must be present and others have a choice of to be present or not.

So number of reflexive relations = 2^{90}.

For a symmetric relation if (a, b) is present then (b, a) is also present (where $a \neq b$). There are 45 such pairs of ordered pairs.

So number of reflexive relations which are also symmetric = 2^{45}

Required number of relations = $2^{90} - 2^{45}$.

[Q.85] In a triangle ABC, the angle bisector BD of $\angle B$ intersects AC in D. Suppose BC = 2, CD = 1 and $BD = \frac{3}{\sqrt{2}}$. The perimeter of the triangle ABC is

[A] $\frac{17}{2}$

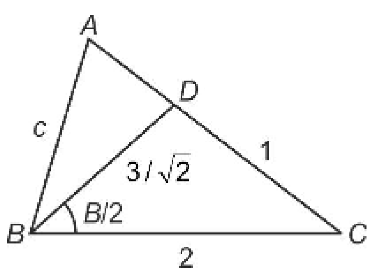
[B] $\frac{15}{2}$

[C] $\frac{17}{4}$

[D] $\frac{15}{4}$

[ANS] **B**

[SOL]



$\because \quad \cos\frac{B}{2} = \frac{\frac{9}{2}+4-1}{6\sqrt{2}} = \frac{5}{4\sqrt{2}}$

Length of angle bisector,

$$BD = \frac{2ac}{a+c}\cos\frac{B}{2}$$

$\Rightarrow \quad \frac{3}{\sqrt{2}} = \left(\frac{4c}{c+2}\right)\cdot\frac{5}{4\sqrt{2}}$

$\Rightarrow \quad c = 3$

We know that $\frac{AB}{BC} = \frac{AD}{CD} \Rightarrow AD = \frac{3}{2}$

Perimeter of $\Delta ABC = 1 + \frac{3}{2} + 3 + 2 = \frac{15}{2}$

[Q.86] Let N be the set of natural numbers. For $n \in N$, define $I_n = \int_0^x \frac{x \sin^{2n}(x)}{\sin^{2n}(x) + \cos^{2n}(x)} dx$. Then for m, $n \in N$

[A] $I_m < I_n$ for all $m < n$

[B] $I_m > I_n$ for all $m < n$

[C] $I_m = I_n$ for all $m \neq n$

[D] $I_m < I_n$ for some $m < n$ and $I_m > I_n$ for some $m < n$

[ANS] **C**

[SOL] $I_n = \int_0^{\pi} \frac{x \cdot \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$ ....(i)

$I_n = \int_0^{\pi} \frac{(\pi - x)\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$ .....(ii)

Adding (i) and (ii), we get

$$2I_n = \pi \int_0^{\pi} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$$

$$\Rightarrow \quad 2I_n = 2\pi \int_0^{\pi/2} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx$$

$$\Rightarrow \quad I_n = \pi \int_0^{\pi/2} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx \quad .....(iii)$$

$$\Rightarrow \quad I_n = \pi \int_0^{\pi/2} \frac{\cos^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx \quad .....(iv)$$

Adding (iii) and (iv), we get

$$2I_n = \pi \int_0^{\pi/2} 1 \cdot dx = \frac{\pi^2}{2}$$

$$\Rightarrow \quad I_n = \frac{\pi^2}{4}$$

I_n is constant for any $n \in N$.

[Q.87] For $\theta \in [0, \pi]$, let $f(\theta) = \sin(\cos\theta)$ and $g(\theta) = \cos(\sin\theta)$. Let $a = \max_{0 \le \theta \le \pi} f(\theta)$, $b = \min_{0 \le \theta \le \pi} f(\theta)$, $c = \max_{0 \le \theta \le \pi} g(\theta)$ and $d = \min_{0 \le \theta \le \pi} g(\theta)$. The correct inequalities satisfied by a, b, c, dare

[A] $b < d < c < a$

[B] d < b < a < c

[C] b < d < a < c

[D] b < a < d < c

[ANS] **C**

[SOL] $\because$ $f'(\theta) = -\cos(\cos\theta)\cdot\sin\theta$

We know that $\cos(\cos\theta) > 0\ \forall\ \theta \in [0, \pi]$ and $\sin\theta \geq 0\ \forall\ \theta \in [0, \pi]$

So, $f(\theta)$ is decreasing function

$a = f(0) = \sin 1$ and $b = f(\pi) = -\sin 1$

$\because$ $g'(\theta) = -\sin(\sin\theta)\cdot\cos\theta$

We know that $\sin(\sin\theta) \geq 0\ \forall\ \theta \in [0, \pi]$

So, $g(\theta)$ is decreasing in $\left(0, \frac{\pi}{2}\right)$ and increasing in $\left(\frac{\pi}{2}, \pi\right)$

$c = \max\{g(0), g(\pi)\} = 1$ and $d = g\left(\frac{\pi}{2}\right) = \cos 1$

Clearly, b < d < a < c

[Q.88] Six consecutive sides of an equiangular octagon are 6, 9, 8, 7, 10, 5 in that order. The integer nearest to the sum of the remaining two sides is

[A] 17

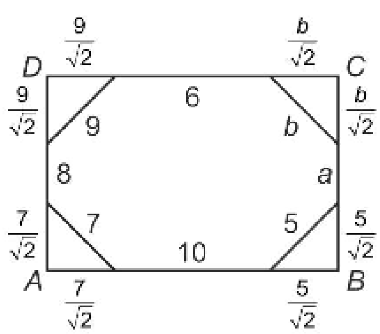
[B] 18

[C] 19

[D] 20

[ANS] **B**

[SOL] Let the remaining two sides be a and b, then



Refer to the diagram,

$\because$ *ABCD* is a rectangle, then

$$\frac{9}{\sqrt{2}}+6+\frac{b}{\sqrt{2}}=\frac{7}{\sqrt{2}}+10+\frac{5}{\sqrt{2}}\Rightarrow\frac{b}{\sqrt{2}}=4+\frac{3}{\sqrt{2}}$$

Similarly, $\frac{9}{\sqrt{2}}+8+\frac{7}{\sqrt{2}}=\frac{b}{\sqrt{2}}+a+\frac{5}{\sqrt{2}}\Rightarrow \quad a=4+\frac{8}{\sqrt{2}}$

Clearly, $a+b=7+8\sqrt{2}=18.3$

[Q.89] The value of the integral $\int_{1}^{\sqrt{2}+1}\left(\frac{x^2-1}{x^2+1}\right)\frac{1}{\sqrt{1+x^4}}dx$ is

[A] $\frac{\pi}{6\sqrt{2}}$

[B] $\frac{\pi}{12\sqrt{2}}$

[C] $\frac{\pi}{8\sqrt{2}}$

[D] $\frac{\pi}{4\sqrt{2}}$

[ANS] **B**

[SOL] $\int_{1}^{\sqrt{2}+1}\frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}}$

Let $x+\frac{1}{x}=t$

$\left(1-\frac{1}{x^2}\right)dx=dt$

$=\int_{2}^{2\sqrt{2}}\frac{dt}{t\sqrt{t^2-2}}$

Let $t=\sqrt{2}z$

$dt=\sqrt{2}dz$

$=\frac{1}{\sqrt{2}}\int_{\sqrt{2}}^{2}\frac{dz}{z\sqrt{z^2-1}}$

$=\frac{1}{\sqrt{2}}\sec^{-1}z\Big|_{\sqrt{2}}^{2}=\frac{1}{\sqrt{2}}\left[\frac{\pi}{3}-\frac{\pi}{4}\right]$

$=\frac{\pi}{12\sqrt{2}}$

[Q.90] Let a = BC, b = CA, c = AB be the side lengths of a triangle ABC, and m be the length of the median through A. If a = 8, b – c = 2, m = 6, then the nearest integer to b is

[A] 7

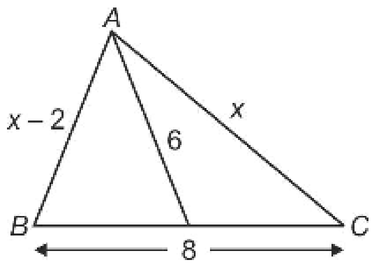
[B] 8

[C] 9

[D] 10

[ANS] **B**

[SOL]



Using Apollonius theorem, we get

$$x^2 + (x-2)^2 = 2[6^2 + 4^2]$$

$\Rightarrow$ $2x^2 - 4x + 4 = 104$

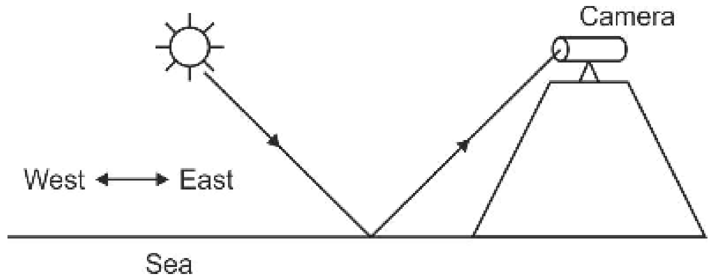
$\Rightarrow$ $x^2 - 2x - 50 = 0$

$\Rightarrow$ $x = 1 + \sqrt{51}$

$\Rightarrow$ The nearest integer to b is 8.

PART-II : PHYSICS

[Q.91] A camera filled with a polarizer is placed on a mountain, in a manner to record only the reflected image of the sun from the surface of a sea as shown in the figure. If the sun rises at 6.00 AM and sets at 6.00 PM during the summer, then at what time in the afternoon will the recorded image have the lowest intensity, assuming there are no clouds and intensity of the sun at the sea surface is constant throughout the day? (Refractive index of water = 1.33)



[A] 12.32 PM

[B] 3.32 PM

[C] 5.00 PM

[D] 6.00 PM

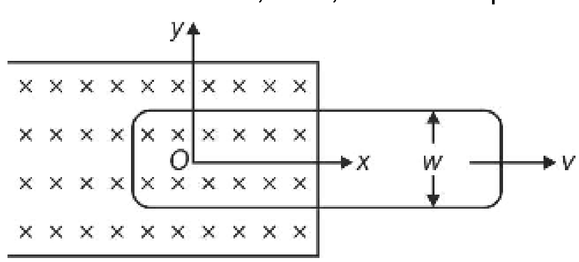
[ANS] **B**

[SOL] Intensity will be minimum when plane polarized light will reach the camera. Reflected light will be completely polarized when angle of incidence is equal to brewster angle, $i = \tan^{-1}\mu = 53°$.

$$\Delta t = \frac{53}{90} \times 6 \times 60 = 212 \text{ minutes after } 12:00 \text{ PM}$$

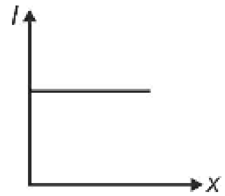
Hence, time at which intensityis minimum = 3:32 PM

[Q.92] Suppose a long rectangular loop of width w is moving along the x-direction with its left arm in a magnetic field perpendicular to the plane of the loop (see figure). The resistance of the loop is zero and it has an inductance L. At time, t = 0, its left arm passes the origin, O.

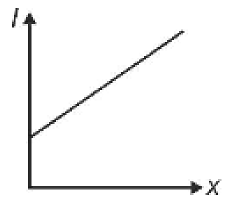


If for $t \geq 0$, the current in the loop is I and the distance of its left arm from the origin is x, then I versus x graph will be

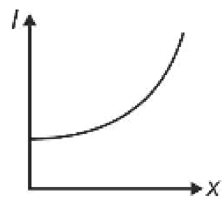
[A]



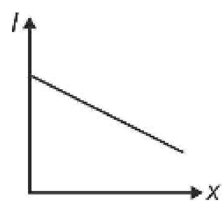
[B]



[C]



[D]



[ANS] **B**

[SOL] $Bwv - L\frac{di}{dt} - iR = 0$

$\Rightarrow$ Ldi = Bwvdt (R = 0)

$\Rightarrow$ $L\int_{i_0}^{i} di = Bw\int_{0}^{x} dx$

$i = i_0 + \frac{Bw}{L}x$

[Q.93] Imagine a world where free magnetic charges exist. In this world, a circuit is made with a U shape wire and a rod free to slide on it. A current carried by free magnetic charges can flow in the circuit. When the circuit is placed in a uniform electric field, E perpendicular to the plane of the circuit and the rod is pulled to the right with a constant speed v, the "magnetic EMF" in the current and the direction of the corresponding current, arising because of changing electric flux will be (ℓ is the length of the rod and c is speed of light)

[A] $vE\ell$ clockwise

[B] vEL counterclockwise

[C] $\frac{vE\ell}{c^2}$ clockwise

[D] $\frac{vE\ell}{c^2}$ counterclockwise

[ANS] **D**

[SOL] Magnetic EMF $=\oint \vec{B}\cdot d\vec{\ell}$

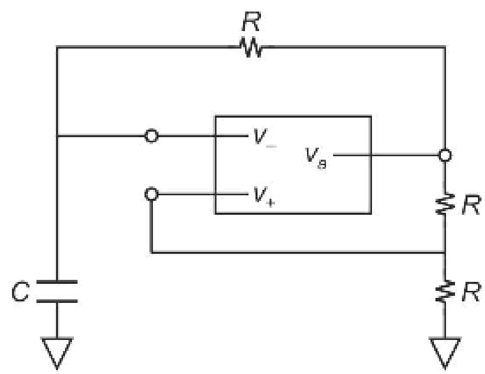
$$=\mu_0\varepsilon_0\frac{d\phi_E}{dt} \text{ (Maxwell-Ampere circuital law)}$$

$$=\mu_0\varepsilon_0 EL\frac{dx}{dt}$$

$$=\frac{EvL}{c^2} \quad \text{counter-clockwise, (Assuming E outwards)}$$

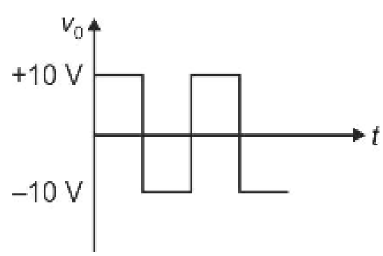
[Q.94] The box in the circuit below has two inputs marked v_+ and v_- and a single output marked V_0.

The output obeys $V_0 = \begin{matrix} +10\text{ V if } v_+ > v_- \\ -10\text{ V if } v_+ < v_- \end{matrix}$

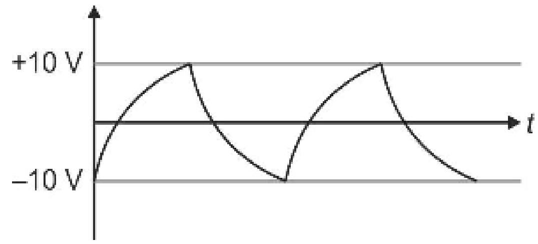


The output V_0 of this circuit a long time after it is switched on is best represented by

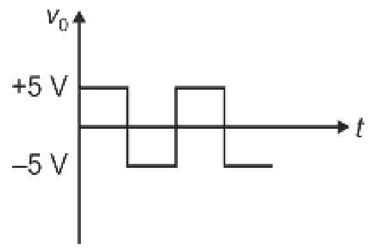
[A]



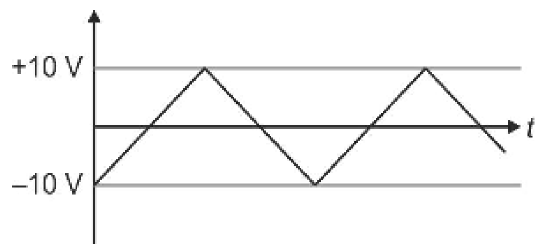
[B]



[C]

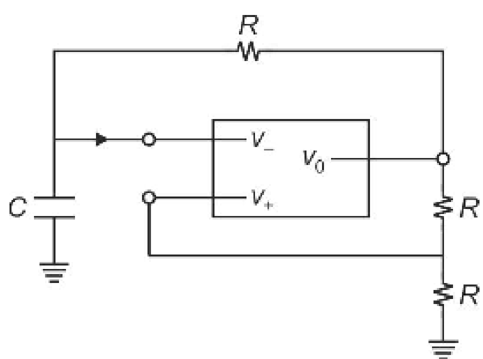


[D]



[ANS] **A**

[SOL]



From the circuit, $v_+ = v_0\left(\frac{R}{R+R}\right) = \frac{v_0}{2}$

Let initially after long time capacitor is fully charged

i.e., $v_c = v_0 \Rightarrow v_- = v_c = v_0$

Now, here given $v_0 = \begin{cases} +10\text{ V if } v_+ > v_- \\ -10\text{ V if } v_+ < v_- \end{cases}$

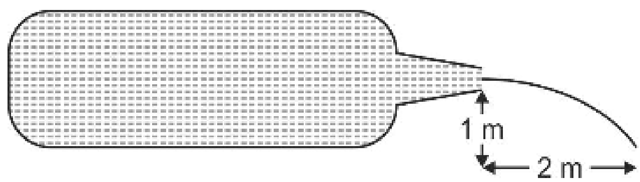
Since $v_- > v_+ \Rightarrow v_0 = -10\text{ V}$

Now, when $v_0 = -10$ V, then $v_+ = \frac{-10}{2} = 5$ V and $v_c = v_- = -10$ V

i.e., $v_+ > v_- \Rightarrow v_0 = +10$ V

Thus, this cycle continues $\Rightarrow$ plot given in option A best represent the situation.

[Q.95] A bottle has a thin nozzle on top. It is filled with water, held horizontally at a height of 1 m and squeezed slowly by hands so that the water jet coming out of the nozzle hits the ground at a distance of 2 m. If the area over which the hands squeeze it is 10 cm^2, the force applied by hand is close to (take g = 10 m/s^2 and density of water = 1000 kg/m^3)



[A] 20 N

[B] 10 N

[C] 5 N

[D] 2.5 N

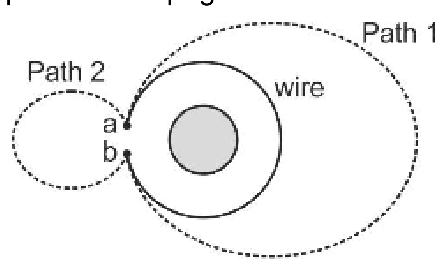
[ANS] **B**

[SOL] $R = v\sqrt{\frac{2h}{g}} \Rightarrow v^2 = \frac{g}{2h} \cdot R^2$, where R = 2 m and h = 1 m

$$\left(P_0 + \frac{F}{A}\right) = P_0 + \frac{1}{2}\rho v^2$$

$$\Rightarrow \quad F = A \cdot \frac{\rho g}{4h} \cdot R^2 = 10 \text{ N}$$

[Q.96] The circular wire in figure below encircles solenoid in which the magnetic flux is increasing at a constant rate out of the plane of the page.



The clockwise emf around the circular loop is ε_0. By definition a voltammeter measures the voltage difference between the two points given by $V_b - V_a = -\int_a^b \bar{E} \cdot d\bar{s}$. We assume that a and b are infinitesimally close to each other. The values of $V_b - V_a$ along the path 1 and $V_a - V_b$ along the path 2, respectively are

[A] $-\varepsilon_0, -\varepsilon_0$

[B] $-\varepsilon_0, 0$

[C] $-\varepsilon_0, \varepsilon_0$

[D] $\varepsilon_0, \varepsilon_0$

[ANS] **B**

[SOL] $\oint \bar{E} \cdot d\vec{s} = -\frac{d\phi}{dt} = \varepsilon_0$

$$V_b - V_a = -\int_a^b \bar{E} \cdot d\vec{s}$$

$V_b - V_a$ for path $1 = -\varepsilon_0$ (flux enclosed is same as loop)

$V_a - V_b$ for path 2 = 0 (flux enclosed is zero)

[Q.97] A beam of neutrons performs circular motion of radius, r = 1 m. under the influence of an inhomogeneous magnetic field with in homogeneity extending over $\Delta r = 0.01$ m. The speed of the neutrons is 54 m/s. The mass and magnetic moment of the neutrons respectively are 1.67×10^{-27} kg and 9.67×10^{-27} J/T. The average variation of the magnetic field over $\Delta r$is approximately

[A] 0.5 T

[B] 1.0 T

[C] 5.0 T

[D] 10.0 T

[ANS] **C**

[SOL] Magnetic dipole placed in non-uniform $\vec{B}$ will experience force

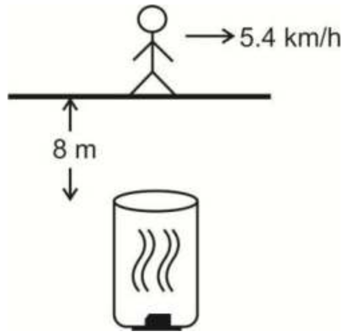
Here, $M\frac{dB}{dr} = \frac{mv^2}{r}$

$$dB = \frac{mv^2}{Mr} \cdot dr$$

$$= \frac{1.67 \times 10^{-27} \times 54^2 \times 0.01}{9.67 \times 10^{-27} \times 1}$$

= 5.04 T

[Q.98] A student is jogging on a straight path with the speed 5.4 km per hour. Perpendicular to the path is kept a pipe with its opening 8 m from the road (see figure). Diameter of the pipe is 0.45 m. At the other end of the pipe is a speaker emitting sound of 1280 Hz towards the opening of the pipes. As the student passes in front of the pipe, she hears the speaker sound for T seconds. T is in the range (Take speed of sound, 320 m/s)



[A] 6 –12

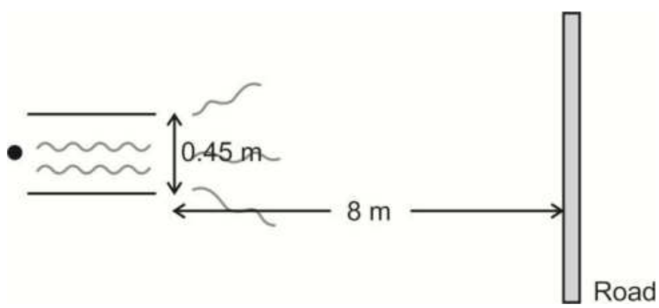
[B] 12 –18

[C] 3 – 6

[D] 18 – 22

[ANS] **A**

[SOL]



First diffraction minima $1.22\frac{\lambda}{a}D = 1.22 \times \frac{320}{1280} \times \frac{8}{0.45} = 5.42$ m

Width of central maxima (W) = 10.84 m

Δt for which sound will be heard $= \frac{W}{v} = 7.23$ sec.

[Q.99] A solar cell is to be fabricated for efficient conversion of solar radiation to emf using material A. The solar cell is to be mechanically protected with the help of a coating using material B. If the band gap energy of materials A and B are E_A and E_B, respectively, then which of the following choices is optimum for better performance of the solar cell.

[A] E_A = 1.5 eV, E_B = 5 eV

[B] E_A = 1.5 eV, E_B = 1.5 eV

[C] $E_A = 3$ eV, $E_B = 1.5$ eV

[D] $E_A = 0.5$ eV, $E_B = 5$ eV

[ANS] **A**

[SOL] Based on fact. Semiconductors with band gap close to 1.5 eV are ideal materials for solar cell fabrication.

[Q.100] The "Kangri" is an earthen pot used to stay warm in Kashmir during the winter months. Assume that the "Kangri" is spherical and of surface area 7×10^{-2} m^2. It contains 300 g of a mixture of coal, wood and leaves with calorific value of 30 kJ/g (and provides heat with 10% efficiency). The surface temperature of the 'kangri' is 60ºC and the room temperature is 0ºC. Then, a reasonable estimate for the duration t (in hours) that the 'kangri' heat will last is (take the 'kangri' to be a black body)

[A] 8

[B] 10

[C] 12

[D] 16

[ANS] **B**

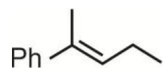
[SOL] Rate of heat emission $= \sigma A(T^4 - T_0^4)$

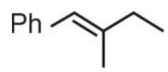
Heat produced due to burning $= \eta \cdot m$ (calorific value)

$$\Delta t = \frac{\text{Heat produced}}{\text{Rate of emission}} = 9.34 \text{ hr}$$

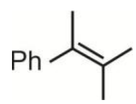
PART-II : CHEMISTRY

[Q.101] An organic compound X with molecular formula $C_{11}H_{14}$ gives an optically active compound on hydrogenation. Upon ozonolysis, X produces a mixture of compounds – P and Q. Compound P gives a yellow precipitate when treated with I_2 and NaOH but does not reduce Tollen's reagent. Compound Q does not give any yellow precipitate with I_2 and NaOH but gives Fehling's test. The compound X is

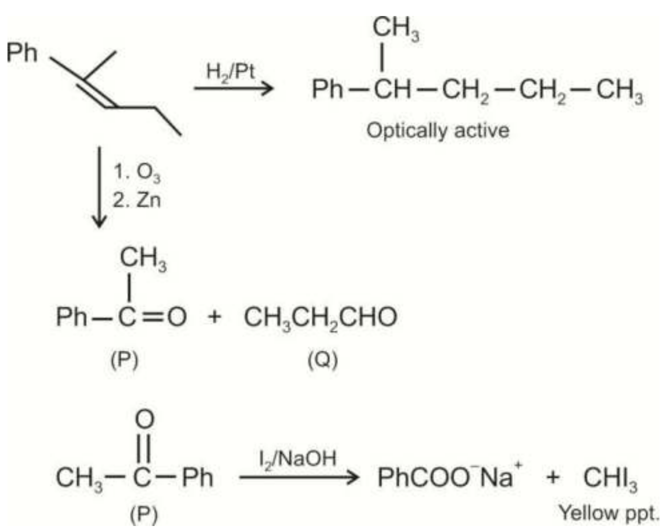
[A] 

[B] 

[C] 

[D] 

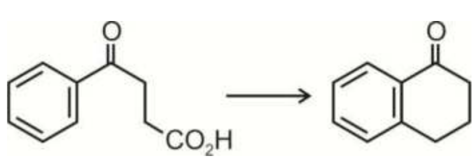
[ANS] **A**

[SOL] 

P is a ketone, will not give Tollen's test

Q is an aliphatic aldehyde, can give Fehling's test

[Q.102] The following transformation



can be carried out in three steps. The reagents required for these three steps in their correct order, are:

[A] (i) $NaBH_4$; (ii) PCl_5; (iii) anh. $AlCl_3$

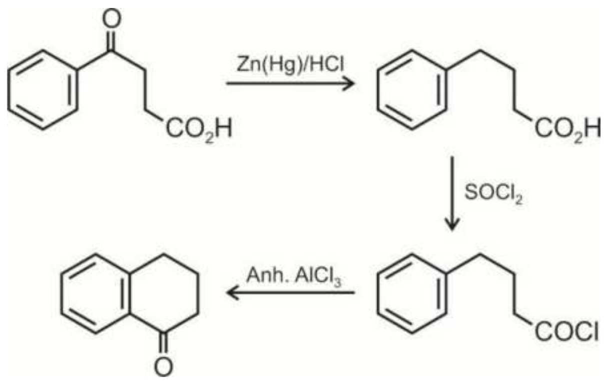
[B] (i) $SOCl_2$; (ii) anh. $AlCl_3$; (iii) Zn(Hg)/HCl

[C] (i) Zn(Hg)/HCl; (ii) $SOCl_2$; (iii) anh. $AlCl_3$

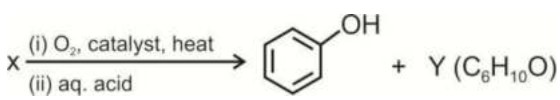
[D] (i) conc. H_2SO_4; (ii) $H_2N–NH_2.H_2O$; (iii) KOH, ethylene glycol, Δ

[ANS] **C**

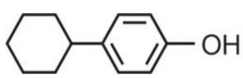
[SOL]

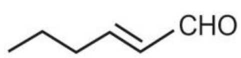


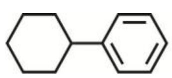
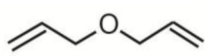
[Q.103] In the following reaction

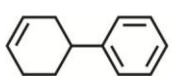
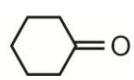


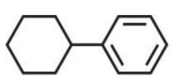
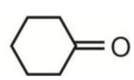
X and Y, respectively, are:

[A] 

and 

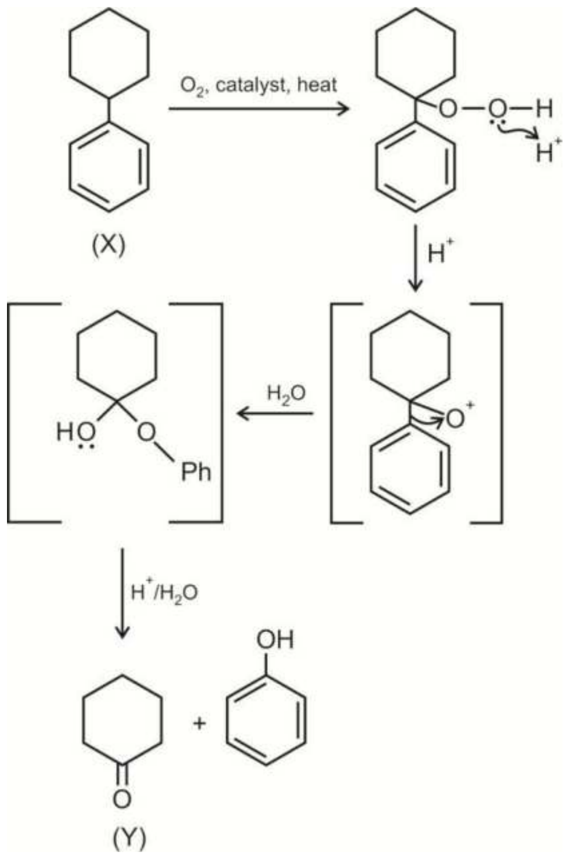
[B]  and 

[C]  and 

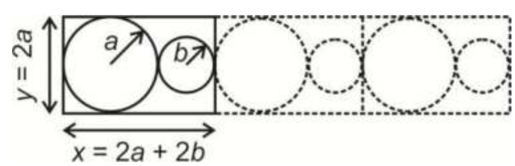
[D]  and 

[ANS] **D**

[SOL]



[Q.104] A two-dimensional solid is made by alternating circles with radius a and b such that the sides of the circles touch. The packing fraction is defined as the ratio of the area under the circles to the area under the rectangle with sides of length x and y.



The ratio r = b/a for which the packing fraction is minimized is closet to

[A] 0.41

[B] 1.0

[C] 0.50

[D] 0.32

[ANS] **A**

[SOL] Packing fraction (PF) $= \dfrac{\pi(a^2+b^2)}{(2a+2b)(2a)}$

$$PF = \frac{\pi a^2\left(1+\left(\frac{b}{a}\right)^2\right)}{4a^2\left(1+\left(\frac{b}{a}\right)\right)} = \frac{\pi(1+r^2)}{4(1+r)};\left[r=\frac{b}{a}\right]$$

$$\frac{d(PF)}{dr} = \frac{\pi}{4}\left[\frac{[(1+r)(2r)-(1+r^2)]}{(1+r)^2}\right] = 0 \text{ if r is minimum}$$

$\therefore$ $2r + 2r^2 - 1 - r^2 = 0$

$r^2 + 2r - 1 = 0$

$$r = \frac{-2\pm\sqrt{4+4}}{2} = \frac{-2+2\sqrt{2}}{2} = \sqrt{2}-1 \simeq 0.41$$

[Q.105] Consider a reaction that is first order in both directions

$$A \underset{k_b}{\overset{k_f}{\rightleftharpoons}} B$$

Initially only A is present, and its concentration is A_0. Assume A_t and A_{eq} are the concentrations of A at time t and at equilibrium, respectively. The time "t" at which $A_t = (A_0 + A_{eq})/2$ is

[A] $t = \dfrac{\ln\left(\frac{3}{2}\right)}{(k_f + k_b)}$

[B] $t = \dfrac{\ln\left(\frac{3}{2}\right)}{(k_f - k_b)}$

[C] $t = \dfrac{\ln 2}{(k_f + k_b)}$

[D] $t = \dfrac{\ln 2}{(k_f - k_b)}$

[ANS] **C**

[SOL] $A \underset{k_b}{\overset{k_f}{\rightleftharpoons}} B$

t = 0 A_0 0

t = t $A_0 - x$ x

t = Equi $A_0 - x_e$ x_e

$$-\frac{d[A]}{dt} = k_f[A] - k_b[B]$$

$$-\frac{d[A]}{dt} = k_f(A_0 - x) - k_b(x)$$

$$-\frac{d[A]}{dt}=k_f(A_0-x_e)-k_b(x_e)=0 \text{ [at equilibrium]}$$

$$k_f(A_0-x_e)=k_b(x_e)$$

$$k_b=\frac{k_f(A_0-x_e)}{x_e}$$

$$+\frac{d[B]}{dt}=k_f(A_0-x)-k_f\frac{(A_0-x_e)}{x_e}(x)$$

$$=\frac{x_ek_f(A_0-x)-k_f(A_0-x_e)x}{x_e}$$

$$+\frac{d[B]}{dt}=k_fA_0\frac{(x_e-x)}{x_e}$$

$$\frac{d[B]}{(x_e-x)}=k_f\frac{A_0}{x_e}dt$$

$$\int_0^x\frac{dx}{(x_e-x)}=\int_0^t\frac{k_fA_0}{x_e}dt$$

$$-[\ln(x_e-x)]_0^x=\frac{k_fA_0}{x_e}t$$

$$\frac{x_e}{A_0t}\ln\frac{x_e}{(x_e-x)}=k_f \qquad ....(1)$$

From old relation

$$k_b=k_f\frac{(A_0-x_e)}{x_e}$$

$$k_bx_e=k_fA_0-k_fx_e$$

$$(k_b+k_f)=k_f\frac{A_0}{x_e} \qquad ....(2)$$

From equation (1) and (2)

$$(k_f+k_b)=\frac{1}{t}\ln\frac{x_e}{(x_e-x)}$$

Now given data

$$\Rightarrow \quad (A_0-x)=A_t$$

$$(A_0-A_t)=x$$

$$\Rightarrow \quad A_t=\frac{A_0+(A_0-x_e)}{2}=\frac{2A_0-x_e}{2}$$

$2A_t = 2A_0 - x_e$

$x_e = 2A_0 - 2A_t$

$(x_6 - x) = 2A_0 - 2A_t - A_0 + A_t$

$(x_e - x) = (A_0 - A_t)$

$$(k_f + k_b) = \frac{1}{t} \ln \frac{x_e}{(x_e - x)}$$

$$t = \frac{1}{(k_f + k_b)} \ln \frac{2A_0 - 2A_t}{(A_0 - A_t)} = \frac{1}{(k_f + k_b)} \ln 2$$

[Q.106] The reaction $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$ is in equilibrium in a closed vessel at 298 K. The partial pressure (in atm) of $CO_2(g)$ in the reaction vessel is closest to:

[Given : the change in Gibbs energies of formation at 298 K and 1 bar for

CaO (s) = –603.501 kJ mol^{-1}

CO_2 (g) = –394.389 kJ mol^{-1}

$CaCO_3$ (s) = –1128.79 kJ mol^{-1}

Gas constant R = 8.314 J K^{-1} mol^{-1}]

[A] 1.13×10^{-23}

[B] 0.95

[C] 1.05

[D] 8.79×10^{23}

[ANS] **A**

[SOL] $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$

$\Delta G^o = \Delta G_f^o(CaO) + \Delta G_f^o(CO_2) - \Delta G_f^o(CaCO_3)$

= – 603.501 – 394.389 + 1128.79

= + 130.9 kJ mol^{-1}

$\Delta G^o = -RT \ln K_P = +130.9 \times 1000 \text{ J mol}^{-1}$

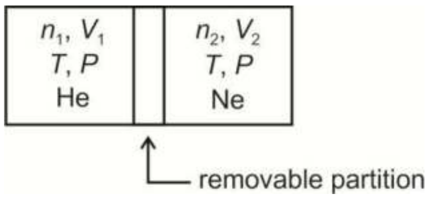
$$\ln K_P = -\frac{130900}{8.314 \times 298} = -52.834$$

$$\log K_P = -\frac{52.834}{2.303} = -22.9414$$

$K_P = 1.13 \times 10^{-23} \text{atm}$

$K_P = P_{CO_2} = 1.13 \times 10^{-23}$ atm

[Q.107] A container is divided into two compartments by a removable partition as shown below:



In the first compartment, n_1 moles of ideal gas He is present in a volume V_1. In the second compartment, n_2 moles of ideal gas Ne is present in a volume V_2. The temperature and pressure in both the compartments are T and P, respectively. Assuming R is the gas constant, the total change in entropy upon removing the partition when the gases mix irreversibly is

[A] $n_1R\ln\frac{V_1}{V_1+V_2}+n_2R\ln\frac{V_2}{V_1+V_2}$

[B] $n_1R\ln\frac{V_1+V_2}{V_1}+n_2R\ln\frac{V_1+V_2}{V_2}$

[C] $(n_1+n_2)R\ln\frac{n_1V_1}{n_2V_2}$

[D] $(n_1+n_2)R\ln\frac{n_2V_2}{n_1V_1}$

[ANS] **B**

[SOL] Entropy change at constant T is given by

$$\Delta S = nR\ln\left[\frac{(V_{final})}{(V_{initial})}\right]$$

ΔS is extensive property hence additive in nature. So total change in entropy is

$$\Delta S_{Total} = n_1R\ln\left(\frac{V_1+V_2}{V_1}\right)+n_2R\ln\left(\frac{V_1+V_2}{V_2}\right)$$

where $(V_1 + V_2)$ is final volume.

[Q.108] Number of stereo isomers possible for the octahedral complexes $[Co(NH_3)_3Cl_3]$ and $[Ni(en)_2Cl_2]$, respectively, are:

[en = 1.2-ethylenediamine]

[A] 2 and 4

[B] 4 and 3

[C] 3 and 2

[D] 2 and 3

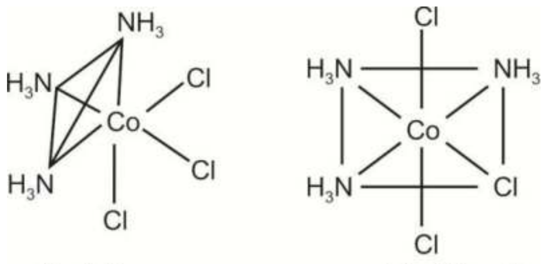
[ANS] **D**

[SOL] $[Co(NH_3)_3Cl_3]$

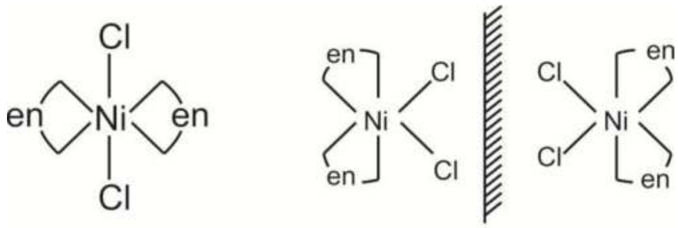
Possible number of stereoisomers of above compound is 2.

$[Ni(en)_2Cl_2]$

Possible number of stereoisomers of above compound is 3.



(Facial) (Meridional)



(trans) (cis) (cis)

[Q.109] When a mixture of NaCl, $K_2Cr_2O_7$ and conc. H_2SO_4 is heated in a dry test tube, a red vapor (X) is evolved. This vapor (X) truns an aqueous solution of NaOH yellow due to the formation of Y. X and Y, respectively, are:

[A] $CrCl_3$ and $Na_2Cr_2O_7$

[B] $CrCl_3$ and Na_2CrO_4

[C] CrO_2Cl_2 and Na_2CrO_4

[D] $Cr_2(SO_4)_3$ and $Na_2Cr_2O_7$

[ANS] **C**

[SOL]

$$4Cl^- + Cr_2O_7^{2-} + 6H^+ \longrightarrow \underset{(X)}{2CrO_2Cl_2} \uparrow + 3H_2O$$

$$CrO_2Cl_2 + 4OH^- \longrightarrow \underset{(Y)}{CrO_4^{2-}} + 2Cl^- + 2H_2O$$

X and Y respectively are CrO_2Cl_2 and Na_2CrO_4.

[Q.110] Sodium borohydride upon treatment with iodine produces a Lewis acid (X), which on heating with ammonia produces a cyclic compound (Y) and a colorless gas (Z), X, Y and Z are:

[A] $X = BH_3$; $Y = BH_3{\cdot}NH_3$; $Z = N_2$

[B] $X = B_2H_6$; $Y = B_3N_3H_6$; $Z = H_2$

[C] $X = B_2H_6$; $Y = B_6H_6$; $Z = H_2$

[D] $X = B_2H_6$; $Y = B_3N_3H_6$; $Z = N_2$

[ANS] **B**

[SOL]

$$2NaBH_4 + I_2 \xrightarrow{\text{in diglyme}} \underset{(x)}{B_2H_6} + H_2 + 2NaI$$

$$3B_2H + 6NH_3 \xrightarrow{\Delta} \underset{(Y)}{2B_3N_3H_6} + \underset{(Z)}{12H_2} \uparrow$$

PART-II : BIOLOGY

[Q.111] Which ONE of the following is the most likely ratio of blood groups (A : B : AB) among the progeny from heterozygous parents with B and AB blood groups?

[A] 0.5 : 0.25 : 0.25

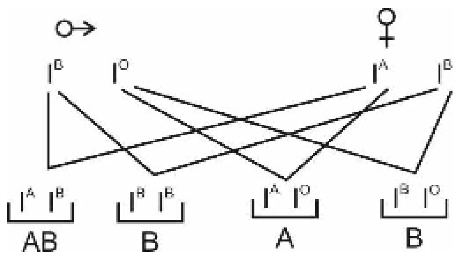
[B] 0.25 : 0.25 : 0.5

[C] 0.25 : 0.5 : 0.25

[D] 0 : 0.25 : 0.75

[ANS] C

[SOL] According to given question;



Hence, number of progeny with blood group A, B and AB are respectively, 1, 2 and 1.

∴ Ratio of blood groups (A : B : AB) among the progenies are

$$=\left(\frac{1}{4}:\frac{2}{4}:\frac{1}{4}\right)=0.25:0.5:0.25$$

[Q.112] Match the plants in Column I with their features listed in **Column II, III & IV.**

Column I	Column II	Column III	Column IV
Types of plants	Types of photosynthesis	Site of Calvin cycle	Time of stomata opening
Rice Pineapple Sugarcan	CAM C4 C3	Mesophyll Bundle sheath	Day Night

Choose the CORRECT combination.

[A] Rice-C3-Mesophyll-Day, Pineapple-CAM-Mesophyll-Night, Sugarcane-C4-Bundle sheath-Day

[B] Rice-C3-Mesophyll-Day, Pineapple-CAM-Mesophyll-Night, Sugarcane-C4-Mesophyll-Day

[C] Rice-C4-Mesophyll-Day, Pineapple-C3-Bundle sheath-Night, Sugarcane-CAM-Bundle sheath-Day

[D] Rice-CAM-Mesophyll-Day, Pineapple-CAM-Mesophyll-Day, Sugarcane-C4-Bundle sheath-Day

[ANS] **A**

[SOL]

Types of Plants	Type of Photosynthesis	Site of Calvin cycle	Time of stomata opening
Rice	C_3	Mesophyll Cell	Day
Pineapple	CAM	Mesophyll Cell	Night
Sugarcane	C_4	Bundle Sheath Cell	Day

[Q.113] A bacteriophage T2 particle contains within its head a double-stranded B-form DNA of molecular weight 1.2×10^8 Da. Assume that the head of a T2 phage particle is of 210 nm in length and the average molecular weight of anucleotide is 330 Da. The length of the T2 genome is in the range of

[A] 6×10^5 to 6.4×10^5 nm

[B] 40×10^4 to 41×10^4 nm

[C] 1.8×10^5 to 2×10^5 nm

[D] 6×10^4 to 6.4×10^4 nm

[ANS] **D**

[SOL] The total weight of B form ds DNA given is

1.2×10^8 Da

The weight of a nucleotide is 330 Da

So, the number of total nucleotides is $\frac{1.2 \times 10^8}{330} = 3.63 \times 10^6$

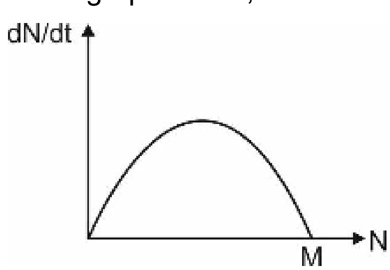
But the genetic material is double stranded so, the genome length is $\frac{3.63 \times 10^6}{2} = 1.8 \times 10^6$.

The distance between two nucleotides in B form DNA is 0.34 nm.

So, the total length of genome is $1.8 \times 10^6 \times 0.34$

Hence, the correct option is D i.e., 6.12×10^4 nm

[Q.114] In the graph below, where N is population size and tis time, M represents



[A] Specific growth rate

[B] Median population size

[C] Carrying capacity

[D] Minimum population size without going extinct

[ANS] **C**

[SOL] According to the given graph, it shows the change in population w.r.t time in parabolic curve.

The graph shows a gradual deceleration. Hence, population density reached the carrying capacity.

Therefore, 'M' represents carrying capacity.

[Q.115] Match the metabolic pathways in **Column I** with their corresponding intermediate molecules listed in **Column II**

Column I	Column II
P. Krebs cycle	i. Dihydroxy acetone phosphate
Q. Glycolysis	ii Succinate
R. Electron transport chain	iii Cytochrome c
S. Nitrogen fixation	iv. Glutamate
	v. Glyoxylate

Choose the CORRECT combination.

[A] P-ii, Q-i, R-iii, S-iv

[B] P-i, Q-v, R-iv, S-ii

[C] P-v, Q-i, R-iii, S-iv

[D] P-ii, Q-i, R-iii, S-v

[ANS] **A**

[SOL] Succinate is an intermediate product of Kreb's cycle.

Dihydroxyacetone phosphate is an intermediate product of glycolysis.

Cytochrome c is mobile electron carrier in electron transport chain.

During nitrogen fixation, ammonia is produced.

Later on, ammonia does not remain in gaseous form in the soil. Hence, protonated to form NH_4^+ ions.

$$NH_3 + H^+ \longrightarrow NH_4^+$$

NH_4^+ is quite toxic to plants and hence, needs to be assimilated to form amino acid in plants.

$$\alpha - \text{Ketoglutaricacid} + NH_4^+ + \text{NADPH} \xrightarrow[\text{dehydrogenase}]{\text{Glutamate}} \text{Glutamate} + H_2O + \text{NADP}$$

Glutamate is produced as a fate of ammonia during reductive amination.

[Q.116] By comparing mitosis and meiosis occurring in the same organism, which ONE of the following options is CORRECT regarding the DNA content per cell?

[A] Mitotic anaphase > Meiotic anaphase I = Meiotic anaphase II

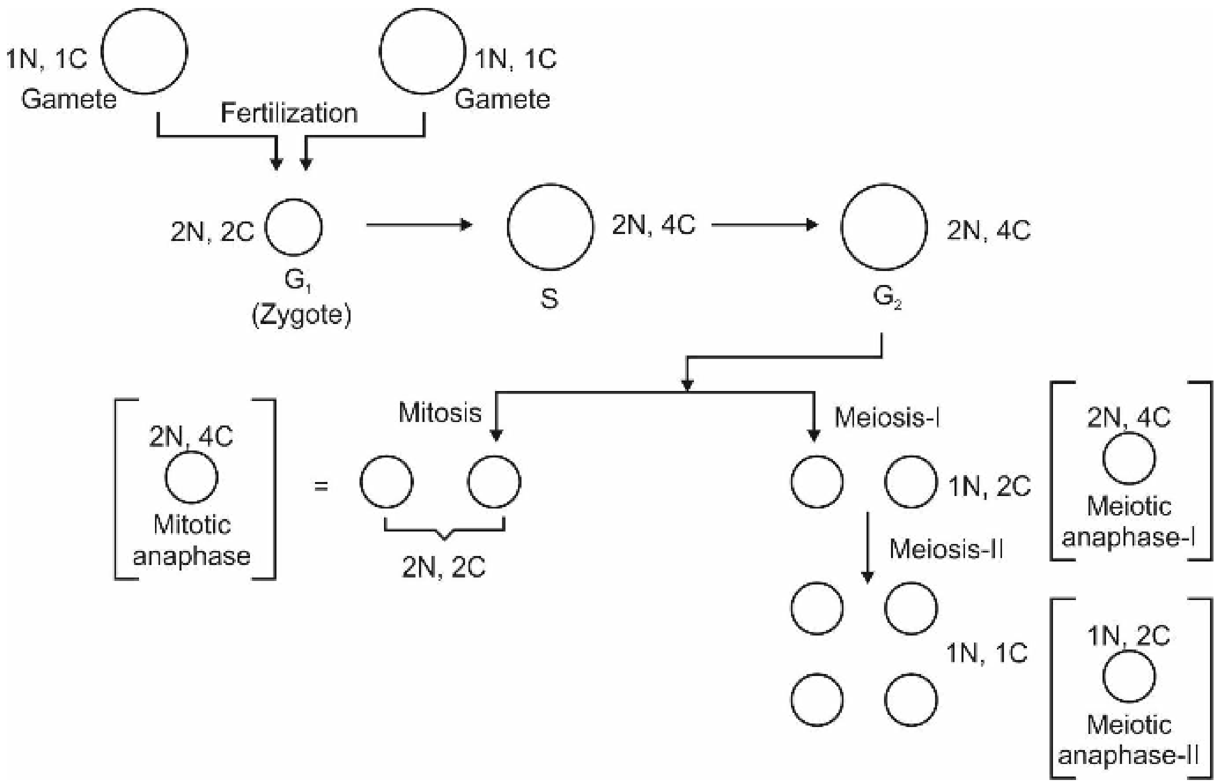
[B] Mitotic anaphase = Meiotic anaphase I > Meiotic anaphase II

[C] Mitotic anaphase < Meiotic anaphase I = Meiotic anaphase II

[D] Mitotic anaphase = Meiotic anaphase I < Meiotic anaphase II

[ANS] **B**

[SOL] According to given question ;



Hence, DNA content in mitotic and meiotic I anaphase are same and greater than meiotic anaphase II stage.

∴ Mitotic anaphase = Meiotic anaphase I > Meiotic anaphase II

[Q.117] Which ONE of the following is likely to occur upon heating a solution of eukaryotic protein from 20°C to 95°C?

[A] Breakage of disulphide bonds

[B] Change in primary structure

[C] Hydrolysis of peptide bonds

[D] Change in tertiary structure

[ANS] **D**

[SOL] Upon heating a solution of eukaryotic protein from 20°C to 95°C, the first evident change will be in the tertiary structure of protein, so, the most relevant answer is option (D).

However, breakage of disulphide bonds, peptide bonds and change in primary structure could be observed in later stages as they involve damage to stronger covalent bonds.

[Q.118] Which ONE of the following statements is INCORRECT about the hexokinase-catalysed reaction given below?

$\text{Glucose} + \text{ATP} \rightarrow \text{Glucose} - 6 - \text{phosphate} + \text{ADP}$

[A] This reaction takes place in the cytoplasm

[B] This is an endergonic reaction

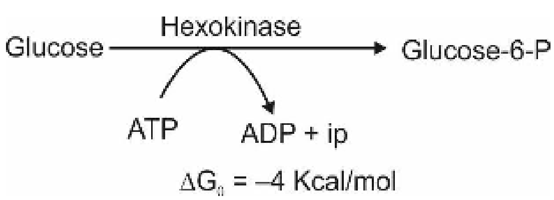
[C] Folding of hexokinase to fit around the glucose molecule excludes water from the active site

[D] This reaction involves an induced fit mechanism in hexokinase

[ANS] **B**

[SOL] $\text{Glucose} + \text{ATP} \rightarrow \text{Glucose} - 6 - \text{phosphate} + \text{ADP}$ is the first reaction of glycolysis, the enzyme hexokinase rapidly phosphorylates glucose entering the cell, forming glucose-6-phosphate (G-6-P).

The overall reaction is exergonic, the free energy change for reaction is -4 Kcal per mole of G-6-P synthesized.



An exergonic reaction is a reaction that releases free energy in the process of reaction.

[Q.119] An ecologist samples trees in multiple forest plots to determine species richness. Which ONE of the following can help determine the adequacy of sampling effort?

[A] Graph the number of new tree species in each successive sampling plot

[B] Graph the total number of tree species per total area for all plots combined

[C] Graph the number of individuals per tree species in each successive sampling plot

[D] 30 sampling plots are sufficient, irrespective of the forest area

[ANS] **A**

[SOL] Species composition is the product of species richness and evenness.

Species richness in a forest can be determined by counting the number of new tree species found in the sampling plot.

[Q.120] In medical diagnostics for a disease, sensitivity (denoted a) of a test refers to the probability that a test result is positive for a person with the disease whereas specificity (denoted b) refers to the probability that a person without the disease tests negative. A diagnostic test for influenza has the values of a = 0.9 and b = 0.9. Assume that the prevalence of influenza in a population is 50%. If a randomly chosen person tests negative, what is the probability that the person actually has influenza?

[A] 0.01

[B] 0.02

[C] 0.05

[D] 0.10

[ANS] **D**

[SOL] P_A = Actually +ve

N_R = Negative report

$\overline{P}_A = \text{Actually} - \text{ve}$

$$P(P_A / N_R) = \frac{P(N_R / P_A) \cdot P(P_A)}{P(N_R / P_A) \cdot P_A + P(N_R / \overline{P}_A) \cdot P(\overline{P}_A)}$$

$$= \frac{\frac{10}{100} \times \frac{50}{100}}{\frac{10}{100} \times \frac{50}{100} + \frac{90}{100} \times \frac{50}{100}} = \frac{500}{5000} = 0.10$$

Correct answer is (D).