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KISHORE VAIGYANIK PROTSAHAN YOJANA – 2020-21
STREAM – SA

Date : 31/01/2021 **Time : 3 hours** **Maximum Marks: 100**

INSTRUCTIONS

The question paper consists of two parts (both contain only multiple choice questions) for 100 marks. There will be four sections in Part I (each section containing 15 questions) and four sections in Part II (each section containing 5 questions)

PART-I

(i) There are 60 objective type questions. **15 questions** from each subject (**Mathematics**, **Physics**, **Chemistry** & **Biology**). All questions are compulsory.

(ii) Each correct answer gets 1 mark and for each incorrect answer 0.25 mark will be deducted.

Part-II

(i) There are 20 objective type questions. **5 questions** from each subject (**Mathematics**, **Physics**, **Chemistry** & **Biology**). All questions are compulsory.

(ii) Each correct answer gets **2 marks** and for each incorrect answer **0.5 mark** will be deducted.

Name of Student :..

Batch : ....

Enrolment No. | | | | | | | | | | | | | | | | |

KISHORE VAIGYANIK PROTSAHAN YOJANA (KVPY) 2020-21

Date of Examination – 31st January, 2021

SOLUTIONS

PART-I : MATHEMATICS

[Q.1] Let [x] be the greatest integer less than or equal to x, for a real number x. Then the equation $[x^2] = x + 1$ has

[A] Two solutions

[B] one solution

[C] No solution

[D] More than two solutions

[ANS] **C**

[SOL] $[x^2] = x + 1$ ...(i)

(i) $\Rightarrow$ x is an integer ...(ii)

$x^2 - \{x^2\} = x + 1$

$x^2 - x - 1 = \{x^2\}$ ...(iii)

(ii) $\Rightarrow x^2 - x - 1 < 1 \quad \Rightarrow -1 < x < 2$ ...(iv)

(ii) & (iv) $\Rightarrow$ possible values of x are 0 & 1.

But 0 and 1 do not satisfy (i).

$\Rightarrow$ No solution

[Q.2] Let $p_1(x) = x^3 - 2020x^2 + b_1x + c_1$ and $p^2(x) = x^3 - 2021x^2 + b_2x + c_2$ be polynomials having two common roots α and β. Suppose there exist polynomials $q_1(x)$ and $q_2(x)$ such that $p_1(x)\, q_1(x) + p_2(x)q_2(x) = x^2 - 3x + 2$. Then the correct identity is

[A] $p_1(3) + p_2(1) + 4028 = 0$

[B] $p_1(3) + p_2(1) + 4026 = 0$

[C] $p_1(2) + p_2(1) + 4028 = 0$

[D] $p_1(1) + p_2(2) + 4028 = 0$

[ANS] **A**

[SOL] Let $p_1(x) = x^3 - 2020x^2 + b_1x + c_1 = (x-\alpha)(x-\beta)(x-\gamma)$

and $p_2(x) = x^3 - 2021x^2 + b_2x + c_2 = (x-\alpha)(x-\beta)(x-\delta)$

$\because\ p_1(x)\cdot q_1(x) + p_2(x)q_2(x) = x^2 - 3x + 2$

Comparing the coefficient of x^3 we get

$q_1(x) = -q_2(x) = q(x)(\text{say})$

So $(x-\alpha)(x-\beta)[q(x)(\delta-\gamma)] = (x-1)(x-2)$

Hence $\alpha = 1, \beta = 2, \gamma = 2017$ and $\delta = 2018$

$p_1(x) = (x-1)(x-2)(x-2017) \Rightarrow p_1(x) = -4028$

$p_2(x) = (x-1)(x-2)(x-2018)$

So $p_1(x) + p_2(1) + 4028 = 0$

[Q.3] Suppose p, q, r are positive rational numbers such that $\sqrt{p}+\sqrt{q}+\sqrt{r}$ is also rational. Then

[A] $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are irrational

[B] $\sqrt{pq}, \sqrt{pr}, \sqrt{qr}$ are rational, but $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are irrational

[C] $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are rational

[D] $\sqrt{pq}, \sqrt{pr}, \sqrt{qr}$ are irrational

[ANS] **C**

[SOL] $\because$ $p, q, r \in Q$ and $\sqrt{p}+\sqrt{q}+\sqrt{r} \in Q$

$\Rightarrow (\sqrt{p}+\sqrt{q}+\sqrt{r})^2 \in Q \Rightarrow \sqrt{pq}+\sqrt{qr}+\sqrt{rp} \in Q$ ...(i)

Case I: Let exactly one of $\sqrt{p}+\sqrt{q}+\sqrt{r}$ is irrational

WLOG, $\sqrt{p} \notin Q$ but $\sqrt{r}, \sqrt{q} \in Q$

From (i), $\sqrt{p}\underbrace{(\sqrt{q}+\sqrt{r})}_{\text{rational}} \in Q$ (contradiction)

Case II: Let exactly two out of $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are irrational.

WLOG, $\sqrt{p}, \sqrt{q} \notin Q$ but $\sqrt{r} \in Q$.

From (i), $\sqrt{p}, \sqrt{q} \notin Q$ but $\sqrt{r} \in Q$.

$\Rightarrow (\sqrt{p}, \sqrt{r})(\sqrt{q}+\sqrt{r}) \in Q$

$\because$ both $\sqrt{p}+\sqrt{r}$ and $\sqrt{q}+\sqrt{r}$ are irrational, hence they must be conjugate of each other. (Contradiction)

Case III: Let all $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are irrational.

Let $\sqrt{p}+\sqrt{q}+\sqrt{r} = x$ when $x \in Q^+$

From (i), $\sqrt{p}(x-\sqrt{p})+\sqrt{q}\sqrt{r} \in Q$

$\Rightarrow x\sqrt{p}+\sqrt{q}\sqrt{r} \in Q$(Contradiction)

Hence, all $\sqrt{p}, \sqrt{q}$ and $\sqrt{r}$ must be rational.

[Q.4] Let A, B, C be three points on a circle of radius 1 such that $\angle ACB = \frac{\pi}{4}$. Then the length of the side AB is

[A] $\sqrt{3}$

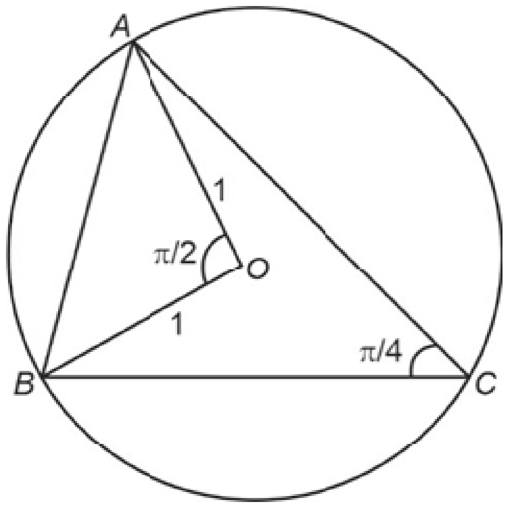
[B] $\frac{4}{3}$

[C] $\frac{3}{\sqrt{2}}$

[D] $\sqrt{2}$

[ANS] **D**

[SOL]



In ΔAOB

$AB^2 = 1^2 + 1^2 = 2$

$AB = \sqrt{2}$

[Q.5] Let x and y be two positive real numbers such that x + y = 1. Then the minimum value of $\frac{1}{x} + \frac{1}{y}$ is

[A] 2

[B] $\frac{5}{2}$

[C] 3

[D] 4

[ANS] **D**

[SOL] $AM \geq HM$

$$\frac{x+y}{2} \geq \frac{2}{\frac{1}{x}+\frac{1}{y}}$$

$$(x+y)\left(\frac{1}{x}+\frac{1}{y}\right) \geq 4$$

$$\frac{1}{x}+\frac{1}{y} \geq 4 \ \ [\because (x+y=1)]$$

[Q.6] Let ABCD be a quadrilateral such that there exists a point E inside the quadrilateral satisfying AE = BE = CE = DE. Suppose $\angle DAB, \angle ABC, \angle BCD$ is an arithmetic progression. Then the median of the set $\{\angle DAB, \angle ABC, \angle BCD\}$ is

[A] $\frac{\pi}{6}$

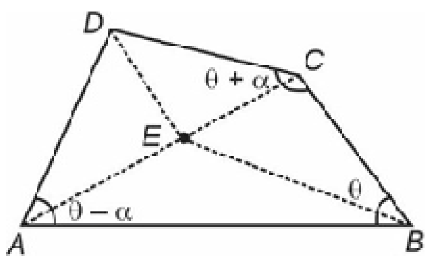
[B] $\frac{\pi}{4}$

[C] $\frac{\pi}{3}$

[D] $\frac{\pi}{2}$

[ANS] **D**

[SOL] $\because$ $\angle DAB, \angle ABC$ and $\angle BCD$ are in A.P.



$\therefore$ Let $\angle DAB = \theta - \alpha, \angle ABC = \theta$ and $\angle BCD = \theta + \alpha$

$\therefore$ Median of $\angle DAB, \angle ABC$ and $\angle BCD = \theta$

From point E all the vertices are at equal distance.

$\therefore$ quadrilateral is cyclic

and $\angle ADC = 2\pi - (\theta - \alpha + \theta + \theta + \alpha) = 2\pi - 3\theta$

and $\angle ADC + \angle ABC = \pi$

$\Rightarrow\ 2\pi - 3\theta + \theta = \pi$

$\Rightarrow\ \theta = \dfrac{\pi}{2}$

[Q.7] The number of ordered pairs (x, y) of positive integers satisfying $2^x + 3^y = 5^{xy}$ is

[A] 1

[B] 2

[C] 5

[D] Infinite

[ANS] **A**

[SOL] $\because\ 2^x + 3^y = 5^{xy}$

When x = y = 1 then 2 + 3 = 5

and $\left(\dfrac{2}{5}\right)^x \cdot \dfrac{1}{5^y} + \left(\dfrac{3}{5}\right)^y \cdot \dfrac{1}{5^x} = 1$ here for any $x, y \in Z^+$ L.H.S. is not equal to 1 as both are less than $\dfrac{1}{2}$.

$\therefore$ Only one ordered pair (1,1) is possible.

[Q.8] If the integers from 1 to 2021 are written as a single integer like 123 … 91011 … 20202021, then the 2021st digit (counted from the left) in the resulting number is

[A] 0

[B] 1

[C] 6

[D] 9

[ANS] **B**

[SOL] Total number of digits used till 99 = 9 + 90 × 2 = 189

The digits use in next 610 three digit numbers = 1830

$\therefore$ Total digit used till the number 710 = 189 + 1830 = 2019

Next three digits are 711

$\therefore$ 2021st digit is 1.

[Q.9] In a triangle ABC, a point D is chosen on BC such that BD : DC = 2 : 5. Let P be a point on the circumcircle ABC such that $\angle PDB = \angle BAC$. Then PD : PC is

[A] $\sqrt{2}:\sqrt{5}$

[B] 2 : 5

[C] 2 : 7

[D] $\sqrt{2}:\sqrt{7}$

[ANS] **D***

[SOL] **?**

[Q.10] Let [x] be the greatest integer less than or equal to x, for a real number x. Then the following sum $\left[\frac{2^{2020}+1}{2^{2018}+1}\right]+\left[\frac{3^{2020}+1}{3^{2018}+1}\right]+\left[\frac{4^{2020}+1}{4^{2018}+1}\right]+\left[\frac{6^{2020}+1}{6^{2018}+1}\right]$ is

[A] 80

[B] 85

[C] 90

[D] 95

[ANS] **B**

[SOL] $\frac{1+x^{2020}}{1+x^{2018}}=\frac{x^2(1+x^{2018})+1-x^2}{1+x^{2018}}=x^2+\frac{1-x^2}{1+x^{2018}}$

Put x = 2 $\left[4+\frac{(-3)}{1+2^{2018}}\right]=3$

Put x = 3 $\left[9+\frac{8}{1+3^{2018}}\right]=8$

Similarly for x = 4 $\left[16-\frac{15}{1+4^{2018}}\right]=15$

For x = 5 $\left[25-\frac{24}{1+5^{2018}}\right]=24$

For x = 6 $\left[36-\frac{35}{1+6^{2018}}\right]=35$

= 85

[Q.11] Let r be the remainder when 2021^{2020} is divided by 2020^2. Then r lies between

[A] 0 and 5

[B] 10 and 15

[C] 20 and 100

[D] 107 and 120

[ANS] **A**

[SOL] $(2021)^{2020} = (1 + 2020)^{2020}$

$= 2020C_0 + 2020C_1 \cdot 2020 + 2020C_2\,(2020)^2 + \ldots$

$= 1 + (2020)^2 + (2020)^2 \times \lambda$, where $\lambda \in z^+$ when divided by (2020) remainder will be 1.

[Q.12] In a triangle ABC, the altitude AD and the median AE divide $\angle A$ into three equal parts. If BC=28, then the nearest integer to AB + AC is

[A] 38

[B] 37

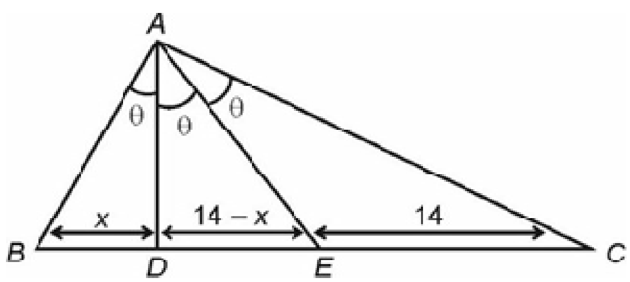
[C] 36

[D] 33

[ANS] **A**

[SOL] Let AD = h and BD = x

In $\Delta ABD : \tan\theta = \frac{x}{h}$ ...(1)



and in $\Delta ADE : \tan\theta = \frac{14-x}{h}$ ...(2)

From equation (1) and (2): $\frac{x}{h} = \frac{14-x}{h} \Rightarrow x = 7$

Now in $\Delta ADC : \tan 2\theta = \frac{28-x}{h}$ and in $\Delta ABD ; \tan\theta = \frac{x}{h}$

$\therefore \frac{\tan 2\theta}{\tan\theta} = \frac{28-x}{x} \quad \Rightarrow 3 - 3\tan^2\theta = 2$

$\therefore \tan^2\theta = \frac{1}{3}$

$\therefore \cos^2\theta = \frac{3}{4}$ and $\sin^2\theta = \frac{1}{4}$

Here AB = $\frac{7}{\sin\theta}$ and $AC = \frac{14}{\tan\theta}$

$$\therefore AB + AC = \frac{7}{\frac{1}{2}} + \frac{14}{\frac{1}{\sqrt{3}}} = 14(1+\sqrt{3}) = 38.248$$

[Q.13] The number of permutations of the letters a_1, a_2, a_3, a_4, a_5 in which the first letter a_1 does not occupy the first position (from the left) and the second letter a_2 does not occupy the second position (from the left) is

[A] 96

[B] 78

[C] 60

[D] 42

[ANS] **B**

[SOL] No. of ways = $5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)+3C_1\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)+3C_2\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right)+3C_3\cdot 1$

= 44 + 27 + 6 + 1 = 78

[Q.14] There are m books in black cover and n books in blue cover, and all books are different. The number of ways these (m + n) books can be arranged on a shelf so that all the books in black cover are put side by side is

[A] m!n!

[B] m!(n + 1)!

[C] (n + 1)!

[D] (m + n)!

[ANS] **B**

[SOL] Bundle m books so total (n + 1) things to arrange Number of ways (n +1)! m!

[Q.15] A 5-digit number $\overline{abcde}$, when multiplied by 9, gives the 5-digit number $\overline{edcba}$. The sum of the digits in thenumber is

[A] 18

[B] 27

[C] 36

[D] 45

[ANS] **B**

[SOL] abcde × 9 = abcde (10 – 1) = abcde0

$$\begin{array}{r} -0abcde \\ \hline 0edcba \end{array}$$

$(a-1)-0=0 \Rightarrow \boxed{a=1}$

$10-e=a \Rightarrow \boxed{e=9}$

$b-a=e \Rightarrow \boxed{b=0}$

$(e-1)-d=b \Rightarrow \boxed{d=8}$

$d-c=c \Rightarrow \boxed{c=4}$

$\overline{abcde}=10989$

sum of digits = 27

PART-I : PHYSICS

[Q.16] A mouse jumps off from the 15th floor of a high-rise building and lands 12 m from the building. Assume that each floor is of 3 m height. The horizontal speed with which the mouse jumps is closest to:

[A] 0

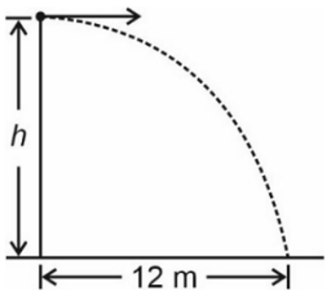
[B] 5 kmph

[C] 10 kmph

[D] 15 kmph

[ANS] **D**

[SOL]



Height of 15th floor, h = 15 × 3 = 45 m

Time taken to reach bottom

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}} = 3\,\text{sec}.$$

Horizontal speed of mouse, $v = \dfrac{\text{Horizontal displacement}}{\text{Time taken}}$

$$\Rightarrow \; v = \frac{12}{3} = 4\,m/s$$

$$\Rightarrow \; v = 14.4\,km/s$$

Which is closest to 15 kmph.

[Q.17] Consider two wires of same material having their ratio of radii to be 2 : 1. If these two wires are stretched by equal force, the ratio of stress produced in them is

[A] $\frac{1}{4}$

[B] $\frac{1}{2}$

[C] $\frac{3}{4}$

[D] 1

[ANS] **A**

[SOL] Given ratio of radii $\frac{r_1}{r_2}=\frac{2}{1}$ ……(A)

Stress $\text{stress}=\frac{\text{force}}{\text{cross-sectional area}}=\frac{F}{A}=\frac{F}{\pi r^2}$

Now the ratio of stress in two wires

$$\frac{\sigma_1}{\sigma_2}=\frac{F/\pi r_1^2}{F/\pi r_2^2}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2}=\frac{r_2^2}{r_1^2}=\left(\frac{r_2}{r_1}\right)^2=\left(\frac{1}{2}\right)^2=\frac{1}{4}$$

[Q.18] A submarine has a window of area 30 × 30 cm^2 on its ceiling and is at a depth of 100 m below sea level in a sea. If the pressure inside the submarine is maintained at the sea-level atmosphere pressure, then the force acting on the window is (consider density of sea water = 1.03 × 10^3 kg/m^3, acceleration due to gravity = 10 m/s^2)

[A] 0.93 × 105 N

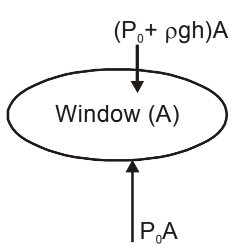
[B] 0.93 × 103 N

[C] 1.86 × 105 N

[D] 1.86 × 103 N

[ANS] **A**

[SOL] Depth of the submarine is 100 m from the sea level and inside pressure of the submarine is atmospheric pressure So net force on the window



Net force on the window = $(P_0+\rho gh)\times A-P_0A=\rho ghA$

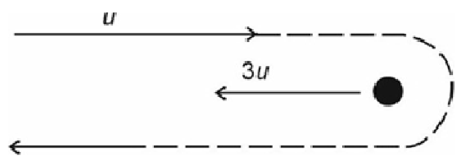
$= 1.03 \times 10^3 \times 10 \times 100 \times 900 \times 10^{-4} N$

$= 1.03 \times 10^4 \times 9N$

$= 9.27 \times 10^4 N$

$\approx 0.93 \times 10^5 N$

[Q.19] A spacecraft which is moving with a speed u relative to the earth in the x-direction, enters the gravitational field of a much more massive planet which is moving with a speed 3u in the negative x-direction. The spacecraft exists following the trajectory as shown below.



The speed of the spacecraft with respect to the earth a long a time after it has escaped the planet's gravity isgiven by

[A] u

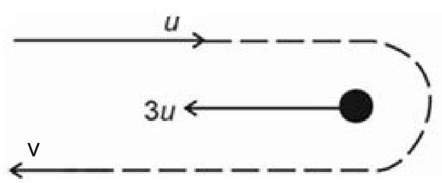
[B] 4u

[C] 2u

[D] 7u

[ANS] **D**

[SOL]



We can consider the case of a head on elastic collision.

$\Rightarrow$ velocity of separation = velocity of approach.

$\Rightarrow v - 3u = u - (-3u)$ (considering large separation between the spacecraft and planet)

$\Rightarrow v = 7u$

[Q.20] The earth's magnetic field was flipped by 180° a million years ago. This flip was relatively rapid and took 10^5 years. Then the average change in orientation per year during the flip was closest to,

[A] 1 second

[B] 5 seconds

[C] 10 seconds

[D] 30 seconds

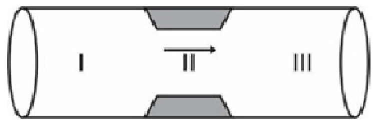
[ANS] **B**

[SOL] Time taken for 180° flip = 10^5 years.

Flip in 1 year = $\frac{180^\circ}{10^5} = \frac{180 \times 60 \times 60}{10^5}$ = 6.48 second

So, closest option is 5 second.

[Q.21] The platelets are drifting with the blood flowing in a streamline flow through a horizontal artery as shown below.



Artery is contracted in region II. Choose the correct statement.

[A] As the platelets enter a constriction, the platelets get squeezed closer together in the narrow region andhence the fluid pressure must rise there

[B] As the platelets enter a constriction, pressure is lower there

[C] The artery's cross section area is smaller in the constriction and thus the pressure must be larger there because pressure equals the force divided by area

[D] Pressure is same in all the parts of the artery

[ANS] **B**

[SOL] Using equation of continuity

$A_1V_1 = A_2V_2$

As, $A_{II} < A_I$

$\Rightarrow V_{II} > V_I$

Now using Bernoulli's theorem

$$\frac{1}{2}\rho V_I^2 + P_I = \frac{1}{2}\rho V_{II}^2 + P_{II}$$

$$\Rightarrow \quad P_I - P_{II} = \frac{1}{2}\rho\left(V_{II}^2 - V_I^2\right) > 0 \qquad (\text{as } V_{II} > V_I)$$

$\Rightarrow$ $P_I > P_{II}$. Thus pressure at the construction is less.

[Q.22] Which of the following colourful patterns is due to diffraction of light?

[A] Rainbow

[B] White light dispersed using a prism

[C] Colours observed on compact disc

[D] Blue colour of sky

[ANS] **C**

[SOL] Reasons (major) for colourful patterns.

(A) Rainbow: Dispersion and Internal reflection

(B) White colour dispersed using a prism: Dispersion

(C) Colours observed on compact disc: Diffraction

(D) Blue colour of sky: Rayleigh scattering

[Q.23] Two balls are projected with the same velocity but with different angles with the horizontal. Their ranges are equal. If the angle of projection of one is 30º and its maximum height is h, then the maximum height of other will be

[A] 1 h

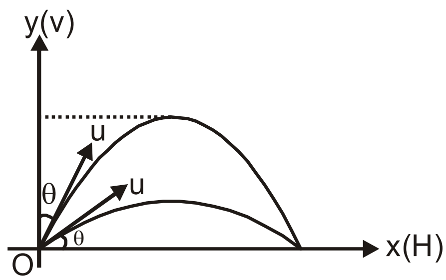
[B] 3 h

[C] 6 h

[D] 10 h

[ANS] **B**

[SOL]



For range to be equal, angles of projection are complementary

$\therefore \quad \theta_1 = 30^\circ$

$\theta_2 = 90^\circ - 30 = 60^\circ$

For first ball

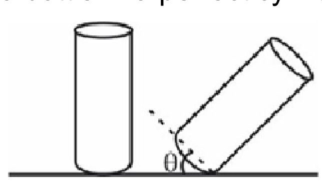
$$h_1 = \frac{u^2 \sin^2(\theta_1)}{2g} = h \Rightarrow \frac{u^2}{8g} = h \qquad \text{.....(A)}$$

For second ball

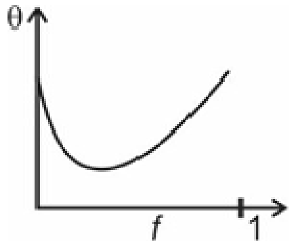
$$h_2 = \frac{u^2 \sin^2(\theta_2)}{2g} = \frac{3u^2}{8g}. \qquad \text{..... (B)}$$

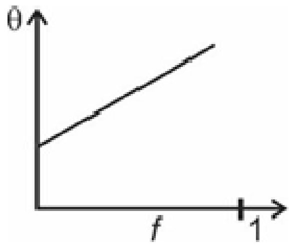
$\Rightarrow h_2 = 3h$

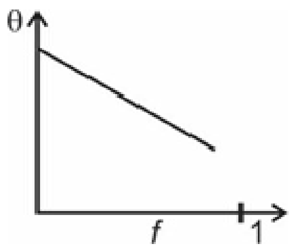
[Q.24] Figure below shows a shampoo bottle in a perfect cylindrical shape.

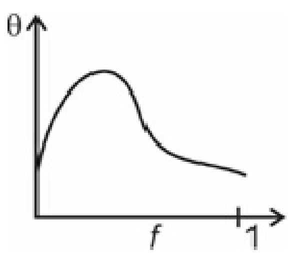


In a simple experiment, the stability of the bottle filled with different amount of shampoo volume is observed. The bottle is tilted from one side and then released. Let the angle θ depicts the critical angular displacement resulting, in the bottle losing its stability and tipping over. Choose the graph correctly depicting the fraction f of shampoo filled (f = 1 corresponds to completely filled) vs the tipping angle θ

[A] 

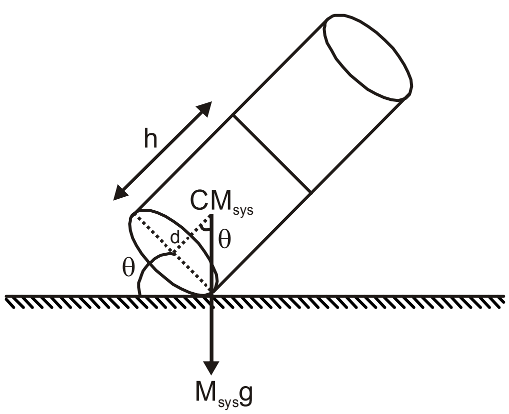
[B] 

[C] 

[D] 

[ANS] **D**

[SOL]

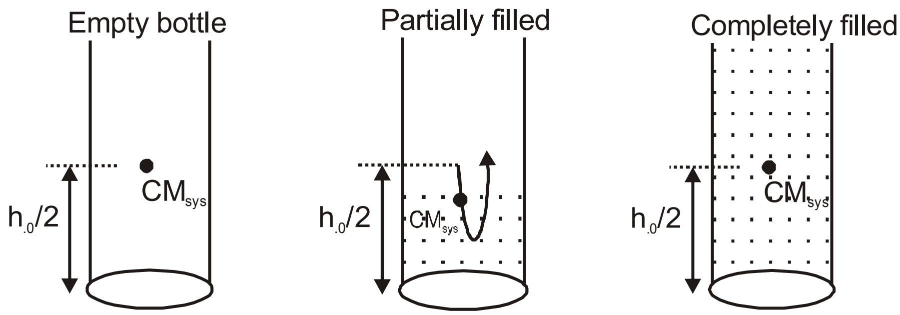


h : height of shampoo, h_0 = height of bottle, R : Radius of bottle

CM_B = CM of bottle, CM_{sh} = CM of shampoo only, CM_{sys} = CM of shampoo + bottle

At tipping angle CM of the system will be just above the contact point of the bottle which the ground.

Thus $\tan\theta = \frac{R}{d}$ .....(A)



In the process of filling the shampoo distance of the CM_{sys}. From the base (d) will at first decrease and then increase up to $\frac{h_0}{2}$.

Accordingly, tipping angle ($\theta = \tan^{-1}\left(\frac{R}{d}\right)$ will at first increase then decrease upto initial value.

So correct option is (D).

[Q.25] At a height of 10 km above the surface of earth, the value of acceleration due to gravity is the same as that of a particular depth below the surface of earth. Assuming uniform mass density for the earth, the depth is,

[A] 1 km

[B] 5 km

[C] 10 km

[D] 20 km

[ANS] **D**

[SOL] $h, d << R$

At the height of h from the earth surface

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

At the depth 'd' from the earth surface.

$$g_d = g\left(1 - \frac{d}{R}\right)$$

As, $g_h = g_d$

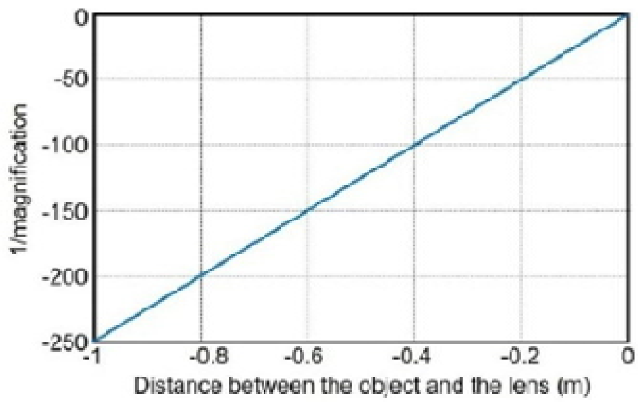
$$g\left(1-\frac{2h}{R}\right)=g\left(1-\frac{d}{R}\right)$$

$$\frac{2h}{R}=\frac{d}{R}$$

d = 2h

= 2 × 10 = 20 km.

[Q.26] The following graph depicts the inverse of magnification versus the distance between the object and lens data for a setup. The focal length of the lens used in the setup is



[A] 250 m

[B] 0.004 m

[C] 125 m

[D] 0.002 m

[ANS] **B**

[SOL] Using lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Multiplying both sides by u.

$$\frac{u}{v}-1=\frac{u}{f}$$

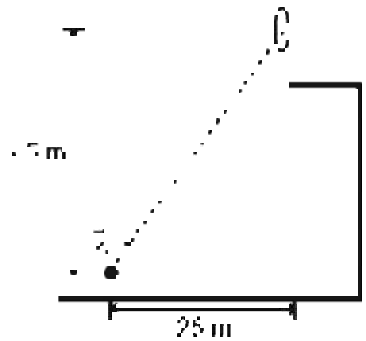
$$\frac{1}{m}=\frac{u}{f}+1 \qquad \left[\text{as}\ \frac{v}{u}=m\right]$$

slope of the given line = $\frac{1}{f}$

$$\Rightarrow \frac{1}{f}=\frac{250-200}{1-0.8}=250$$

$$\Rightarrow f=\frac{1}{250}=0.004\,\text{m s.}$$

[Q.27] In a circus, a performer throws an apple towards a hoop held at 45 m height by another performer standing on a high platform (see figure below). The thrower aims for the hoop and throws the apple with a speed of 24 m/s. At the exact moment that the thrower releases the apple, the other performer drops the hoop. The hoop falls straight down. At what height above the ground does the apple go through the hoop?



[A] 21 m

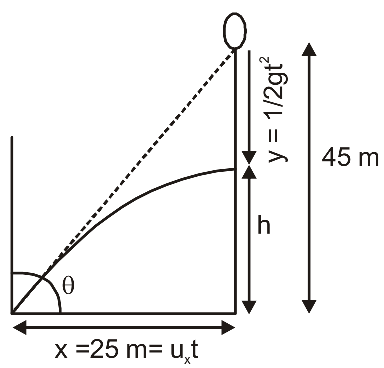
[B] 22 m

[C] 23 m

[D] 24 m

[ANS] **B**

[SOL]



$$\tan\theta = \frac{45}{25} = \frac{9}{5}$$

$$\cos\theta = \frac{5}{\sqrt{106}}$$

Time of crossing the hoop by the apple,

$$t = \frac{x}{u_x} = \frac{25}{24\cos\theta} = \frac{25}{24 \times 5/\sqrt{106}} \quad .....(A)$$

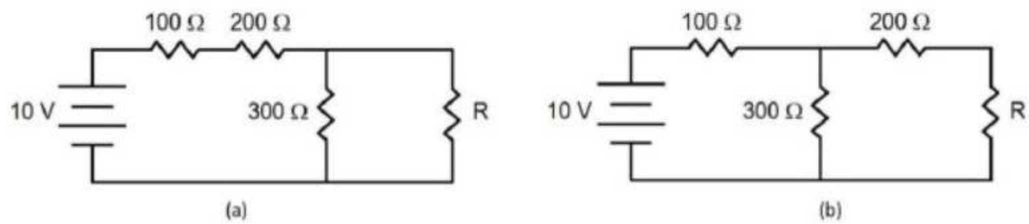
$$h = 45 - \frac{1}{2}gt^2$$

$$= 45 - \frac{10}{2} \times \frac{25 \times 25 \times 106}{24 \times 24 \times 5 \times 5}$$

$$h = 45 - \frac{5 \times 25 \times 106}{576}$$

h = 22 m

[Q.28] A student was trying to construct the circuit shown in the figure below marked (a), but ended up constructing the circuit marked (b). Realizing her mistake, she corrected the circuit, but to her surprise, the output voltage (across *R*) did not change.



The value of resistance *R* is

[A] 100 Ω

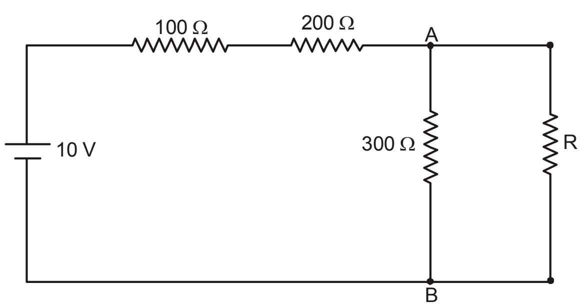
[B] 150 Ω

[C] 200 Ω

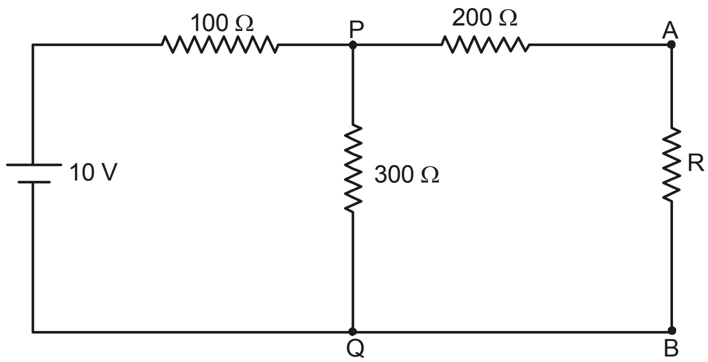
[D] 300 Ω

[ANS] **A**

[SOL] **(a)**



$$V_{R_1} = 10 \times \frac{\frac{300R}{300+R}}{(100+200) + \frac{300R}{300+R}} \qquad .....(A)$$



$$V_{AB} = V_{PQ} \times \frac{R}{R+200}$$

$$V_{R_2} = V_{AB} = 10 \times \frac{300 \times (200+R)/(300+(200+R))}{100 + \left[300 \times (200+R)/(300(200+R))\right]} \times \frac{R}{R+200} \quad \text{.....(B)}$$

From A and B

$$10 \times \frac{\frac{300R}{300+R}}{(100+200) + \frac{300R}{300+R}} = 10 \times \frac{300 \times (200+R)/(300+(200+R))}{100 + \left[300 \times (200+R)/(300(200+R))\right]} \times \frac{R}{R+200}$$

$$\Rightarrow \frac{R}{300(300+R)+300R} = \frac{200+R}{100(500+R)+300(200+R)} \times \frac{R}{R+200}$$

$$\Rightarrow 300(300+R)+300R = \left[100(500+R)+300(200+R)\right]$$

$$\Rightarrow 3(300+R)+3R = 50+R+3(200+R)$$

$$\Rightarrow 900+6R = 1100+4R$$

$$\Rightarrow 2R = 200 \Rightarrow R = 100\Omega$$

[Q.29] The ratio of gravitational force and electrostatic repulsive force between two electrons is approximately (gravitational constant = 6.7×10^{-11} Nm^2/Kg^2, mass of an electron = 9.1×10^{-31} kg, charge on an electron = 1.6×10^{19} C)

[A] 24×10^{-24}

[B] 24×10^{-36}

[C] 24×10^{-44}

[D] 24×10^{-54}

[ANS] **C**

[SOL] $$F_G = \frac{Gm_1m_2}{r^2}$$

$$= \frac{6.7 \times 10^{-11} \times 9.1 \times 10^{-31} \times 9.1 \times 10^{-31}}{r^2}$$

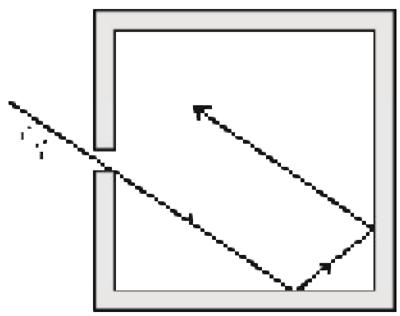
$$F_E = \frac{k\,q_1 q_2}{r^2}$$

$$= \frac{9\times10^{9}\times1.6\times10^{-19}\times1.6\times10^{-19}}{r^2}$$

$$\frac{F_G}{F_E} = \frac{6.7\times(9.1)^2\times10^{-73}}{9\times(1.6)^2\times10^{-29}}$$

$$\frac{F_G}{F_E} = \frac{554.8\times10^{-44}}{23.04} = 24\times10^{-44}$$

[Q.30] A monochromatic beam of light enters a square enclosure with mirrored interior surfaces at an angle of incidence $\theta_i (\neq 0)$ (see the figure below). For some value(s) of θ_i, the beam is reflected by every mirrored wall (other than the one with the opening) exactly once and exits the enclosure through the same hole. Which of the following statements about this beam is correct?



[A] The beam will not come out of the enclosure for any value of θ_i

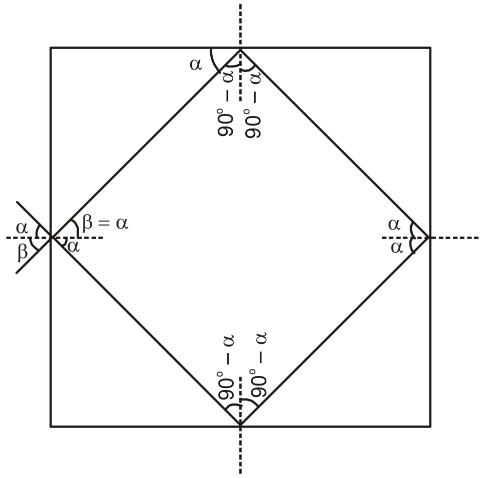
[B] The beam will come out for more than two values of θ_i

[C] The beam will come out only at $\theta_i = 45^\circ$

[D] The beam will come out for exactly two values of θ_i

[ANS] **C**

[SOL]



As per the drawn diagram the ray comes out from square $\beta = \alpha$

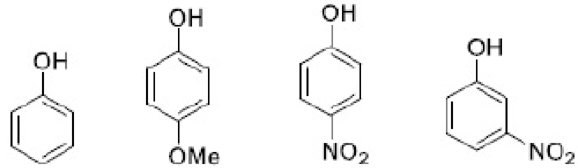
From the symmetry in above diagram, the reflected rays form the square.

$2\alpha = 90°$

$\alpha = 45°$

PART-I : CHEMISTRY

[Q.31] The acidity of



follows the order

[A] I > II > III > IV

[B] IV > III > II > I

[C] III > IV > I > II

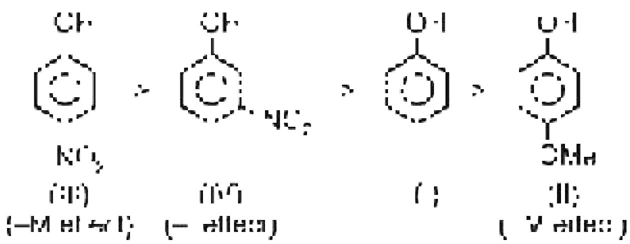
[D] III > II > IV > I

[ANS] **C**

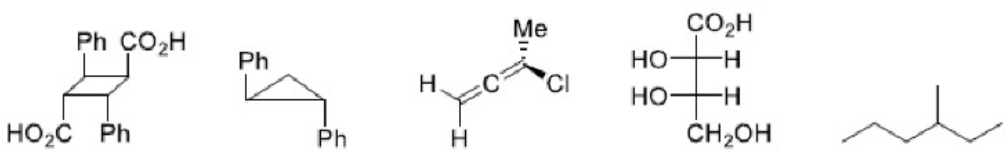
[SOL] Electron withdrawing graphs (–M effect, –I effect) present on phenol increases its acidic nature.

Electron donating groups (+M effect) present on phenol decreases its acidic nature.

Acidic nature:



[Q.32] Among the following,



the compounds which can exhibit optical activity are:

[A] Only II, IV and V

[B] Only IV and V

[C] Only I, II and V

[D] Only I, II and IV

[ANS] **A**

[SOL] Compounds which do not have any type of symmetry exhibit optical activity.

(I) has centre of symmetry so optically inactive

(III) has plane of symmetry so optically inactive

(II), (IV) and (V) do not have any type of symmetry so exhibit optical activity

[Q.32] A molecule which has 1°, 2° and 3° carbon atoms is:

[A] 2,3,4-trimethylpentane

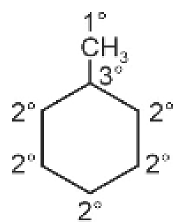
[B] chlorocyclohexane

[C] 2,2-dimethylcyclohexane

[D] methylcyclohexane

[ANS] **D**

[SOL] Methyl cyclohexane:



Number of 1° carbon atoms = 1

Number of 2° carbon atoms = 5

Number of 3° carbon atoms = 1

[Q.34] The organic compound which can be purified by steam distillation is

[A] Acetone

[B] Aniline

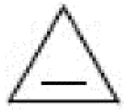
[C] Glucose

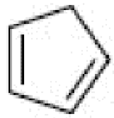
[D] Ethanol

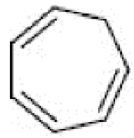
[ANS] **B**

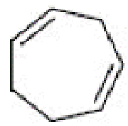
[SOL] Steam distillation method can be used for the purification of impure sample of aniline.

[Q.35] Among the following, the most acidic compound is

[A] 

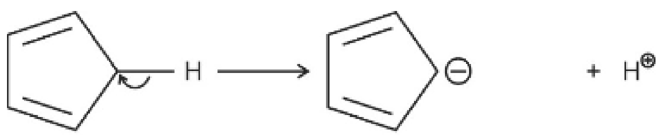
[B] 

[C] 

[D] 

[ANS] **B**

[SOL] More the stability of conjugate base formed by the hydrocarbon, more will be the acidic nature of hydrocarbon.



[Q.36] A closed 10 L vessel contains 1 L water gas (1 : 1 CO : H_2) and 9 L air (20% O_2 by volume) at STP. The contents of the vessel are ignited. The number of moles of CO_2 in the vessel is closest to:

[A] 0.22

[B] 0.022

[C] 0.90

[D] 3.60

[ANS] **B**

[SOL] 1 L water gas has 1 : 1 CO and H_2 gases

So, $V_{H_2} = V_{CO} = 0.5\text{ L}$

$$V_{O_2} = \frac{9 \times 20}{100} = 1.8\text{ L}$$

Now, on ignition

$$\underset{0.5\text{ L}}{2CO(g)} + \underset{1.8\text{ L}}{O_2(g)} \rightarrow \underset{0.5\text{ L}}{2CO_2(g)} \quad \text{at STP}$$

$\therefore$ Number of moles of CO_2 formed $= \frac{0.5}{22.4} = 0.022$ moles

[Q.37] A certain metal has a work function of Φ = 2 eV. It is irradiated first with 1 W of 400 nm light and later with 1 W of 800 nm light. Among the following, the correct statement is:

[Given: Planck constant (h) = 6.626×10^{-34} m^2 kg s^{-1} ; Speed of light (c) = 3×10^8 m s^{-1}]

[A] Both colors of light give rise to same number of photoelectrons.

[B] 400 nm light gives rise to less energetic photoelectrons than 800 nm light.

[C] 400 nm light leads to more photoelectrons.

[D] 800 nm light leads to more photoelectrons.

[ANS] **C**

[SOL] Energy associated with 1 W of 400 nm light

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}} J$$

$= 3.1$ eV

Likely, energy associated with 1 W of 800 nm light

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{800 \times 10^{-9}} 1.5 \text{ eV}$$

Since, work function of the metal = 2 eV

Therefore, only 400 nm light gives rise to ejection of photoelectrons.

[Q.38] Among the following, the correct statement about the chemical equilibrium is:

[A] Equilibrium constant is independent of temperature.

[B] Equilibrium constant is tells us how fast the reaction reaches equilibrium.

[C] At equilibrium, the forward and the backward reactions stop so that the concentrations of reactants and products are constant.

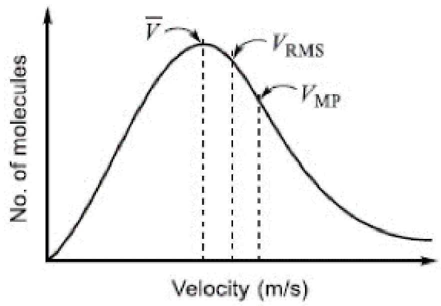
[D] Equilibrium constant is independent of whether you start the reaction with reactants or products.

[ANS] **D**

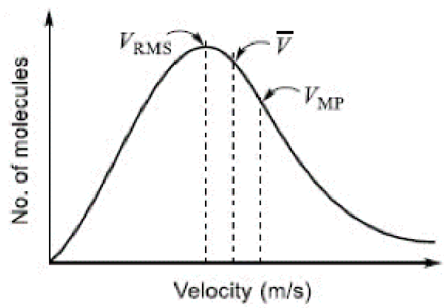
[SOL] Equilibrium constant depends on temperature but it does not tell about the rate of a reaction. At equilibrium, the forward and the backward reactions move with the same rate therefore concentration of all the species becomes constant.

Equilibrium constant (K_{eq}) value is independent of whether reaction has been started with reactants or products.

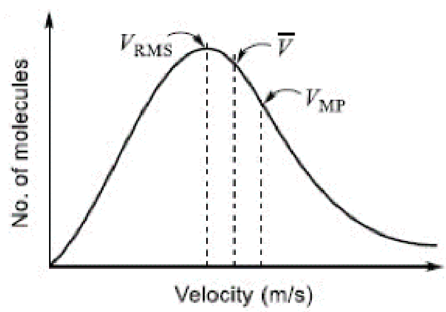
[Q.39] Among the following, the plot that shows the correct marking of most probable velocity (V_{MP}), average velocity ($\overline{V}$), and root mean square velocity (V_{RMS}) is:



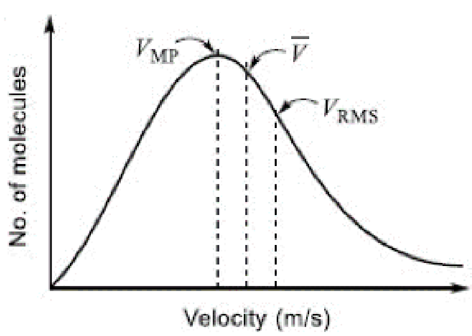
[A]



[B]



[C]



[D]

[ANS] **D**

[SOL] $V_{RMS} = \sqrt{\frac{3RT}{M}}$

$\bar{V} = \sqrt{\frac{8RT}{\pi M}}$

$V_{mp} = \sqrt{\frac{2RT}{M}}$

So, decreasing order of various molecular speeds is, as

$V_{RMS} > \bar{V} > V_{MP}$

[Q.40] The correct set of quantum numbers for the unpaired electron of Cu atom is :

[A] $n = 3, l = 2, m = -2, s = +\frac{1}{2}$

[B] $n = 3, l = 2, m = +2, s = -\frac{1}{2}$

[C] $n = 4, l = 0, m = 0, s = +\frac{1}{2}$

[D] $n = 4, l = 1, m = +1, s = +\frac{1}{2}$

[ANS] **C**

[SOL] Cu(29); $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^1, 3d^{10}$

So, for the unpaired electron of Cu that is present in $4s^1$, set of quantum numbers is;

$n = 4, l = 0, m = 0, s = \pm\frac{1}{2}$

$\therefore$ Correct option is (C)

[Q.41] Among the following, the most polar molecule is :

[A] $AlCl_3$

[B] CCl_4

[C] $SeCl_6$

[D] $AsCl_3$

[ANS] **D**

[SOL] • $AlCl_3$, CCl_4 and $SeCl_6$ are non-polar molecules since dipole moments of these are zero.

- Dipole moment of $AsCl_3$ is non-zero, so it is a polar molecule.

[Q.42] The covalent characters of $CaCl_2$, $BaCl_2$, $SrCl_2$ and $MgCl_2$ follow the order:

[A] $CaCl_2 < BaCl_2 < SrCl_2 < MgCl_2$

[B] $BaCl_2 < SrCl_2 < CaCl_2 < MgCl_2$

[C] $CaCl_2 < BaCl_2 < MgCl_2 < SrCl_2$

[D] $SrCl_2 < MgCl_2 < CaCl_2 < BaCl_2$

[ANS] **B**

[SOL]
- With increase in size of cation, their polarizing power decreases hence covalent character of chlorides also decreases.
- So, correct order of covalent characters is $BaCl_2 < SrCl_2 < CaCl_2 < MgCl_2$

[Q.43] Among the following, the correct statement is:

[A] 100. has four significant figures

[B] 1.00×10^2 has four significant figures

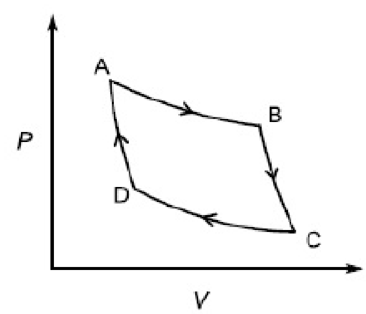
[C] 2.005 has four significant figures

[D] 0.0025 has four significant figures

[ANS] **C**

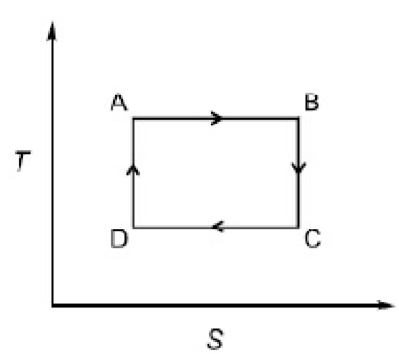
[SOL] Zero between two non-zero digits are significant. So, 2.005 has four significant figures.

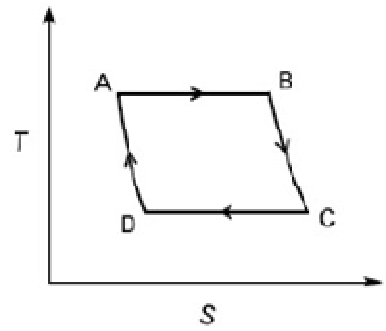
[Q.44] A thermodynamic cycle in the pressure (P) – volume (V) plane is given below:

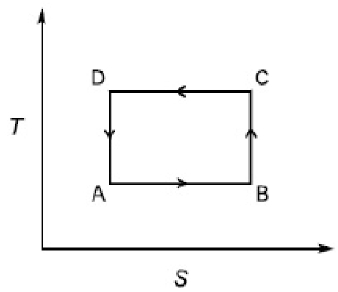


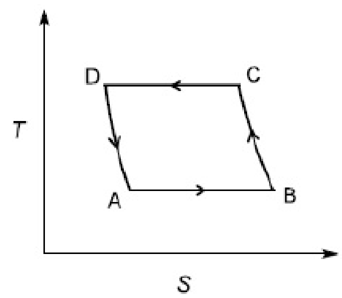
AB and CD are isothermal processes while BC and DA are adiabatic processes. The same cycle in the temperature (T) – entropy (S) plane is:

[A]



[B] 

[C] 

[D] 

[ANS] **A**

[SOL] AB $\rightarrow$
- Isothermal expansion (i.e. $T_A = T_B$)
- On moving A to B, volume increases and P decreases so, entropy will increase.

BC $\rightarrow$
- Adiabatic expansion, so cooling occur.
  So $T_B > T_C$
- For adiabatic process, $\Delta S = 0$, means entropy remains same.

CD $\rightarrow$
- Isothermal compression (i.e. $T_C = T_D$)
- On moving C to D, volume decreases and P increases so, entropy will decrease.

DA $\rightarrow$
- Adiabatic compression, so heating occur.
  So $T_A > T_D$
- For adiabatic process, $\Delta S = 0$ means entropy remains same

[Q.45] The first ionization potential (IP) of the elements Na, Mg, Si, P, Cl and Ar are 5, 14, 7.65, 8.15, 10.49, 12.97 and 15.76 eV, respectively. The IP (in eV) of K is closest to:

[A] 13.3

[B] 18.2

[C] 4.3

[D] 6.4

[ANS] **C**

[SOL] On moving down in a group, ionization potential generally decreases. So, ionization potential of K should be less than that of Na.

PART-I : BIOLOGY

[Q.46] Which ONE of the following chemicals serves as a substrate for carbonic anhydrase?

[A] O_2

[B] CO_2

[C] NO_2

[D] CO

[ANS] **B**

[SOL] The correct answer is option (B) because carbonic anhydrase facilitates the following reaction in both directions during transport of CO_2.

$$\underset{\text{(Substrate)}}{CO_2} + H_2O \rightleftharpoons H_2CO_3 \rightleftharpoons HCO_3^- + H^+$$

[Q.47] Which One of the following is NOT a function of the small intestine ?

[A] Absorption of end products of digestion

[B] Digestion of proteins

[C] Digestion of lipids

[D] Acidification of ingested food

[ANS] **D**

[SOL] Small intestine is responsible for digestion of carbohydrates, proteins and lipids. It is also responsible for absorption of end products of digestion such as monosaccharides, fatty acids, glycerol, monoglycerides, amino acids etc.

Acidification of digested food occurs in stomach due to secretion of HCI from oxyntic cells of gastric glands.

[Q.48] Insulin stimulates the conversion of glucose to

[A] fructose

[B] glycogen

[C] sucrose

[D] starch

[ANS] **B**

[SOL] The correct answer is option (B) because insulin is a hypoglycemic hormone and stimulates conversion of glucose to glycogen (glycogenesis) in the target cells.

[Q.49] Which ONE of the following statements about ecosystem energetics is INCORRECT?

[A] The metabolic requirements of poikilotherms are higher than that of homeotherms.

[B] Autotrophs form the base of the food chain in natural ecosystems.

[C] In terrestrial ecosystems, most of the primary production is consumed by detritivores and not herbivores.

[D] Approximately 10% energy of one trophic level is transferred to the next level.

[ANS] A

[SOL] Poikilotherms also called ectotherms are cold blooded organisms. They cannot regulate their body temperature. Since thermoregulation is energetically expensive, so metabolism of homeotherms is faster. Thus homeotherms have high metabolic requirements than poikilotherms.

[Q.50] Proton motive force is created by pumping protons across the

[A] trans-Golgi network

[B] endoplasmic reticulum

[C] mitochondrial inner membrane

[D] early endosomal membrane

[ANS] C

[SOL] Chemiosmotic theory explains the formation of ATP in chloroplast and mitochondria. During mitochondrial ETS, proton motive force is created by pumping protons across the mitochondrial inner membrane.

[Q.51] Which ONE of the following Mendelian diseases is an example of X-linked recessive disorder?

[A] Haemophilia

[B] Phenylketonuria

[C] Sickle cell anaemia

[D] Beta-thalassemia

[ANS] A

[SOL] Haemophilia is a X-linked recessive disorder. Phenylketonuria, sickle cell anaemia and thalassemia are autosomal recessive disorders.

[Q.52] Which ONE of the following pairs gives rise to fruit and seed, respectively, in a typical angiosperm plant?

[A] Ovule and ovary

[B] Ovary and pollen

[C] Pollen and anther

[D] Ovary and ovule

[ANS] D

[SOL] In a typical angiospermic plant fruit and seeds are formed by ovary and ovule respectively.

[Q.53] The concept of vaccination arose from Edward Jenner's observation that

[A] Injecting inactivated anthrax spores in sheep protected them from anthrax

[B] Injecting humans with tuberculosis-infected lung extracts protected them from tuberculosis

[C] Milk-maids previously infected with cowpox did not contract small pox

[D] Injecting inactivated rabies virus in humans protected them from rabies

[ANS] **C**

[SOL] The correct answer is (C) because Edward Jenner observed that milk-maids previously infected with cowpox did not contract small pox. This is a case of active immunity, infection of cowpox virus (similar to small pox virus) in milk-maids stimulated the immune system of milk-maids against small pox virus.

[Q.54] A plant with genotype AABBCC is crossed with another plant with aabbcc genotype. How many different genotypes of pollens is possible in an F_1 plant if these three loci follow independent assortment?

[A] 8

[B] 4

[C] 2

[D] 1

[ANS] **A**

[SOL]

P_1 P_2

AABBCC aabbcc

↓

F_1 AaBbCc (Here n, heterozygous locus = 3)

Gametes = $2^n = (2)^3 = 8$

In F_1 the pollen as well as egg will be of 8 different types.

[Q.55] Which ONE of the following sequences of events CORRECTLY represents mitosis?

[A] Metaphase, telophase, prophase, anaphase

[B] Anaphase, prophase, metaphase, telophase

[C] Prophase. anaphase, metaphase, telophase

[D] Prophase, metaphase, anaphase, telophase

[ANS] **D**

[SOL] The correct sequence of events of mitosis is

Prophase $\rightarrow$ Metaphase $\rightarrow$ Anaphase $\rightarrow$ Telophase.

[Q.56] The amount of air that is left behind in lungs after expiratory reserve volume has been exhaled is

[A] Inspiratory reserve volume

[B] Tidal volume

[C] Residual volume

[D] Vital capacity

[ANS] **C**

[SOL] The air left in lungs after exhalation of expiratory reserve volume is residual volume

FRC = ERV + RV

So, RV = FRC – ERV

[Q.57] Match the species in **Column I** with their respective feature of body organisation in **Column II**.

	Column I		**Column II**
P.	Mollusca	i.	Pseudocoelom
Q.	Annelida	ii.	Radula
R.	Nematoda	iii.	Radial symmetry
S.	Echinodermata	iv.	Segmentation

Choose the CORRECT combination.

[A] P-ii, Q-i, R-iv, S-iii

[B] P-ii, Q-iv, R-i, S-iii

[C] P-iii, Q-iv, R-i, S-ii

[D] P-iv, Q-iii, R-ii, S-i

[ANS] **B**

[SOL] Molluscs have rasping organ called Radula. Annelids represent presence of metameric segmentation. Pseudocoelom is characteristically present in nematodes. Radial symmetry is present in adult echinoderms.

[Q.58] Who among the following scientists proposed the theory of natural selection independently of Charles Darwin?

[A] Alfred Russel Wallace

[B] Carl Linnaeus

[C] Georges Cuvier

[D] Jean-Baptiste Lamarck

[ANS] **A**

[SOL] The correct answer is (A) because Alfred Russel Wallace who worked in Malay Archipelago had also come to similar conclusions as that of Charles Darwin for natural selection around the same time independently.

[Q.59] The maximum concentration of harmful chemicals is expected to be found in organisms

[A] At the bottom of a food chain.

[B] At the middle of a food chain.

[C] At the top of a food chain.

[D] At any level in a food chain.

[ANS] C

[SOL] In biomagnification the harmful chemicals get accumulated in tissues in increasing concentrations along the food chain. The maximum concentration is found in top consumers that occupy top of food chain.

[Q.60] The genome of SARS-CoV2 is composed of

[A] Double stranded DNA

[B] Double stranded RNA

[C] Single stranded DNA

[D] Single stranded RNA

[ANS] D

[SOL] Severe acute respiratory syndrome corona virus 2 (SARS-CoV2) contains single stranded RNA molecule as its genetic material.

PART-II : MATHEMATICS

[Q.61] Let A denote the set of all 4–digit natural numbers with no digit being 0. Let $B \subset A$ consist of all numbers x such that no permutation of the digits of x gives a number that is divisible by 4. Then the probability of drawing a number from B with all even digits is

[A] $\frac{625}{1641}$

[B] $\frac{16}{641}$

[C] $\frac{16}{1641}$

[D] $\frac{1000}{1641}$

[ANS] **C**

[SOL] For number not divisible by 4 and not having zero can be formed as

Number of ways to form

All digits odd = 5^4

3 digits odd + 1even (4 or 8) = $5^3.2.4$

2 digits odd + 2 even (4 or 8) = 0

1 digit odd + 3 even = 0

All even = 2^4 (Only 2 or 6 can be used)

Probability $= \frac{2^4}{5^4 + 5^3.8 + 2^4} = \frac{16}{1641}$.

[Q.62] Let ABC be a triangle such that AB = 4, BC = 5 and CA = 6. Choose points D, E, F on AB, BC, CA respectively, such that AD = 2, BE = 3, CF = 4. Then $\frac{\text{area}\,\Delta DEF}{\text{area}\,\Delta ABC}$

[A] $\frac{1}{4}$

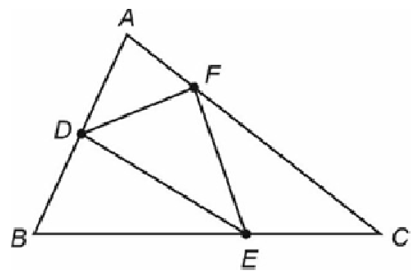
[B] $\frac{3}{15}$

[C] $\frac{4}{15}$

[D] $\frac{7}{30}$

[ANS] **C**

[SOL]



Here a = 5, b = 6 and c = 4

Area of $\Delta ADF = \frac{1}{2}.2.2\sin A = 2\sin A$

Area of $\Delta BDF = \frac{1}{2}.2.3\sin B = 3\sin B$

and area of $\Delta CEF = \frac{1}{2}.2.4\sin C = 4\sin C$

and area of $\Delta ABC = \Delta$

$$\therefore \frac{\text{area of } \Delta DEF}{\text{area of } \Delta ABC} = \frac{\Delta - (2\sin A + 3\sin B + 4\sin C)}{\Delta}$$

$$= 1 - \frac{1}{\Delta}\left(2.\frac{2\Delta}{bc} + 3.\frac{2\Delta}{ac} + 4.\frac{2\Delta}{ab}\right)$$

$$= 1 - \frac{2(2a + 3b + 4c)}{abc}$$

$$= 1 - \frac{2(10 + 18 + 16)}{5.6.4} = \frac{16}{60} = \frac{4}{15}.$$

[Q.63] The number of ordered pairs (x, y) of integers satisfying $x^3 + y^3 = 65$ is

[A] 0

[B] 2

[C] 4

[D] 6

[ANS] **B**

[SOL] $x^3 + y^3 = 65$

Let $x, y > 0$

Clearly (1, 4) and (4, 1) holds for x or $y \geq 6$ difference of two cubes is always greater than or equal to 91.

Hence only 2 ordered pair possible.

[Q.64] A bottle in the shape of a right–circular cone with height h contains some water. When its base is placed on a flat surface, the height of the vertex from the water level is a units. When it is kept upside down, the height of the base from the water level is a/4 units. Then the ratio h/a is.

[A] $\frac{1+\sqrt{85}}{4}$

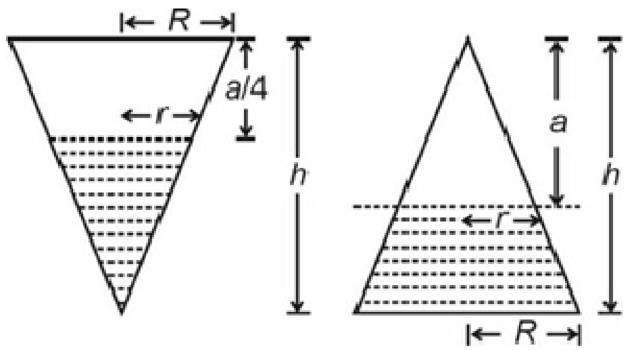
[B] $\frac{1+\sqrt{85}}{8}$

[C] $\frac{1+\sqrt{65}}{4}$

[D] $\frac{1+\sqrt{65}}{8}$

[ANS] **B**

[SOL]



Volume of water in both the cases will be equal i.e.,

$$\frac{1}{3}\pi r_1^2\left(h-\frac{a}{4}\right)=\frac{1}{3}\pi R^2 h-\frac{1}{3}\pi r^2 a \quad \ldots (1)$$

$$\frac{r}{a}=\frac{R}{h} \quad \ldots (2)$$

$$r_1 h = R\left(h-\frac{a}{4}\right) \quad \ldots (3)$$

By (1) and (2) and (3), we get

$$16\left(\frac{h}{a}\right)^2-4\left(\frac{h}{a}\right)-21=0 \Rightarrow \frac{h}{a}=\frac{1+\sqrt{85}}{8}.$$

[Q.65] Consider the following two statements :

I. If n is a composite number, then n divides (n – 1) !

II. There are infinitely many natural numbers n such that $n^3 + 2n^2 + n$ divides n!

[A] I and II are true

[B] I and II are false

[C] I is true and II is false

[D] I is false and II is true

[ANS] **D**

[SOL] I does not hold for n = 4

$\Rightarrow$ I is false

For statement II

$n(n+1)^1 \mid n!$

$\Rightarrow \ (n+1)^2 \mid (n-1)!$

Let $n = 3k-1,\ k > 3, k \in N$

$n+1 = 3k, n-1 = 3k-2$

$(n-1)! = (3k-2)!$

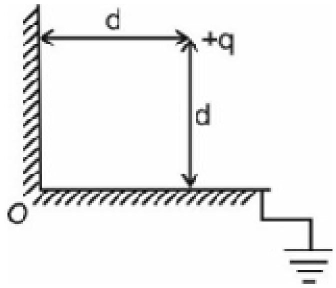
$= (3k-2)\times(3(k-1))\times\ldots\times(2k+1)(2k)(2k-1)\ldots\times(k+1)k(k-1)\ldots3\times2\times1$

RHS contains 3^2, k^2 hence is divisible by $(3k)^2$.

$\Rightarrow \ (n-1)!$ is divisible by $(n+1)^2 \Rightarrow$ II is true.

PART-II : PHYSICS

[Q.66] A charge +q is situated at a distance 'd' away from both the sides of a grounded conducting 'L' shaped sheet as shown in the figure.



The force acting on the charge +q is

[A] Towards O, magnitude $\frac{q^2}{32\pi\varepsilon_0 d^2}\left(2\sqrt{2}+1\right)$

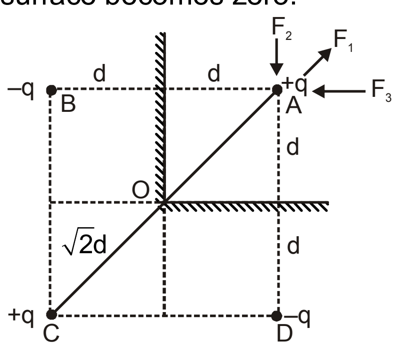
[B] Away from O, magnitude $\frac{q^2}{32\pi\varepsilon_0 d^2}\left(2\sqrt{2}+1\right)$

[C] Towards O, magnitude $\frac{q^2}{32\pi\varepsilon_0 d^2}\left(2\sqrt{2}-1\right)$

[D] Away from O, magnitude $\frac{q^2}{32\pi\varepsilon_0 d^2}\left(2\sqrt{2}-1\right)$

[ANS] **C**

[SOL] Applying image method in which additional charges are placed so that potential of earthed surface becomes zero.



$F_2 = \frac{Kq^2}{4d^2}$ (attractive)

$F_3 = \frac{Kq^2}{4d^2}$ (attractive)

$F_1 = \frac{Kq^2}{\left(2\sqrt{2}d\right)}$ (repulsion)

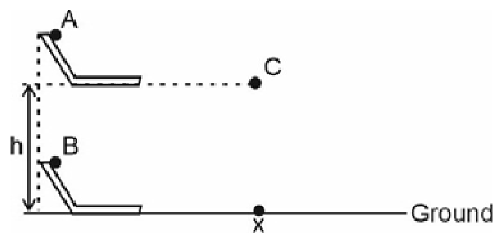
Solving F_{net} along AC

$$F_{net} = \sqrt{2}F - F_1 = \frac{\sqrt{2}Kq^2}{4d^2} - \frac{Kq^2}{8d^2}$$

$$=\left(2\sqrt{2}-1\right)\frac{Kq^2}{8d^2}$$

$$=\frac{q^2\left(2\sqrt{2}-1\right)}{32\pi\varepsilon_0 d^2}, \text{ towards O.}$$

[Q.67] Three balls, A, B and C are release and all reach the point X (shown in the figure). Balls A and B are released from two identical structures, one kept on the ground and the other at height h, from the ground as shown in the figure. They take time t_A and t_B respectively to each X (time starts after they leave the end of the horizontal portion of the structure). The ball C is released from a point at height, h, vertically above X and reaches X in time t_C. Choose the correct statement.



[A] $t_C < t_A < t_C$

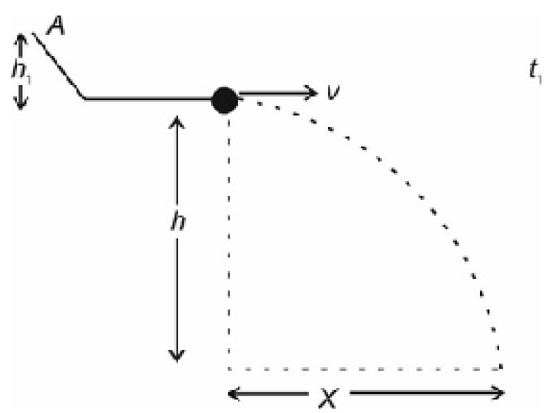
[B] $t_C = t_A = t_C$

[C] $t_C = t_A < t_C$

[D] $t_B < t_A = t_C$

[ANS] **B**

[SOL] **Dimensions of angular structure is negligible** $(h_1 << h)$



For ball A

As angular structure in the case of A and B are identical thus horizontal velocity imparted to A and B will be equal.

As, horizontal displacement of A and B are also same thus time taken by A and B to reach at x will be same. Thus

$t_A = t_B$ ....(A)

As vertical component of initial velocities of A and C are same and equal to zero. Thus time taken by A and C to reach the ground will be same and equal to $\sqrt{\frac{2h}{g}}$.

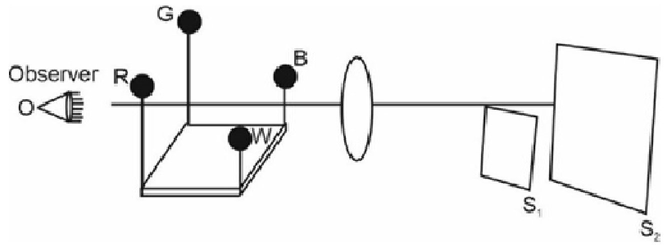
Thus $t_A = t_C$ .....(B)

From equation (A) and (B)

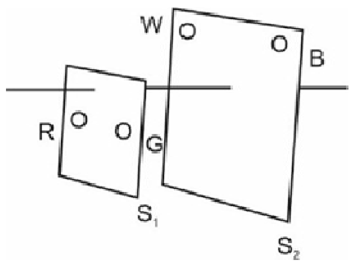
$t_A = t_B = t_C$

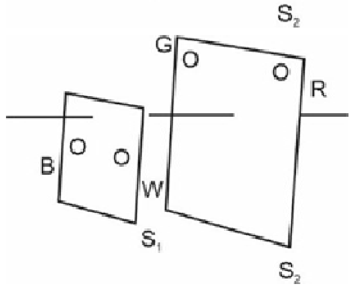
In option (B), it should be $t_A = t_B = t_C$

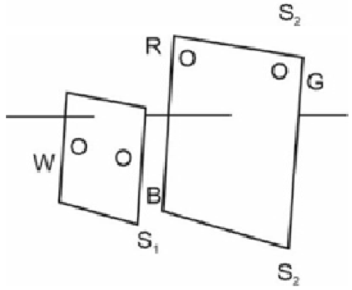
[Q.68] Four bulbs : red, green, white and blue (denoted by R, G, W and B respectively) are kept in front of a converging lens (as shown in the figure below). The observer sees that the green and blue bulbs are kept to the left of the principle axis while the red and white bulbs are kept to the right of the principle axis. He also sees that the red S1 and S2 are set appropriate positions for the focusing to view the images.

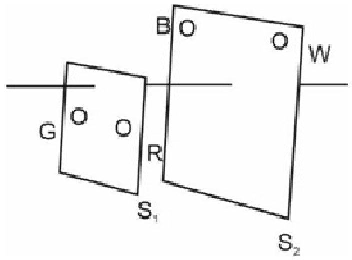


Choose the figure that correctly represents the images as seen by the observer.

[A] 

[B] 

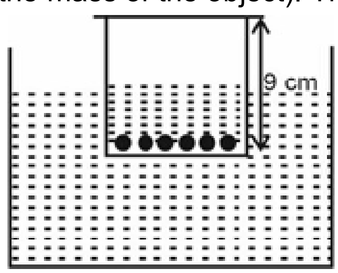
[C] 

[D] 

[ANS] **A**

[SOL] (a) Real images formed by converging lens are inverted.
Thus,
(i) images of bulb on left of principal axis should form on right of principal axis and vice-versa.
(ii) Images of bulb on top of principal axis should be below the principal axis and vice-versa.
(b) In the case of real images by converging lens near is the object to the focus, further will be its image from the focus.

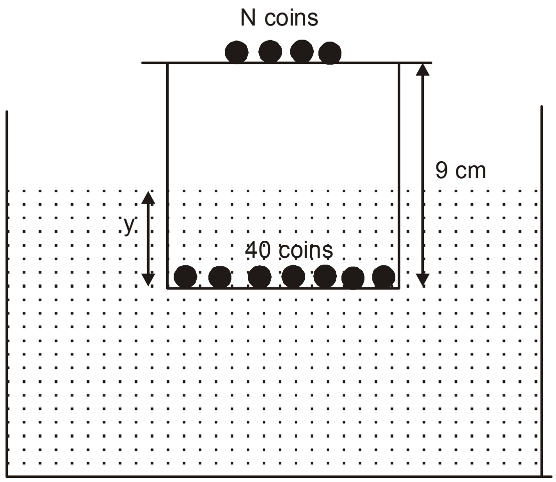
[Q.69] A wide bottom cylindrical massless plastic container of height 9 cm has 9 cm has 40 identical coins inside it and is floating on water with 3 cm inside the water. If we start putting more of such coins on its lid, it is observed that after N coins are put, its equilibrium changes from stable to unstable. Equilibrium in floating is stable if the geometric centre of the submerged portion is above the centre of the mass of the object). The value of N is closed to



[A] 6
[B] 10
[C] 16
[D] 24

[ANS] **B**

[SOL]



Weight/Height of water displaced $\propto$ number of total coins

$$\Rightarrow y \Rightarrow \frac{y}{3\,\text{cm}} = \frac{40+N}{40}$$

$$\Rightarrow y = \frac{3(40+N)}{40} \qquad \ldots\ldots(A)$$

Centre of displaced water from the bottom of the container

$$y_w = \frac{y}{2} = \frac{3(40+N)}{80} \qquad \ldots\ldots(B)$$

Centre of mass of the system from the bottom of the container

$$y_c = \frac{40\times 0 + N\times 9}{40+N} = \frac{9N}{40+N} \qquad \ldots\ldots(C)$$

For stable equilibrium

Centre of the displaced water shall be higher than the centre of mass

$$\Rightarrow y_w > y_c$$

$$\Rightarrow \frac{3(40+N)}{80} > \frac{9N}{40+N}$$

$$\Rightarrow (40+N)^2 > 240N$$

$$\Rightarrow N^2 + 80N + 1600 > 240N$$

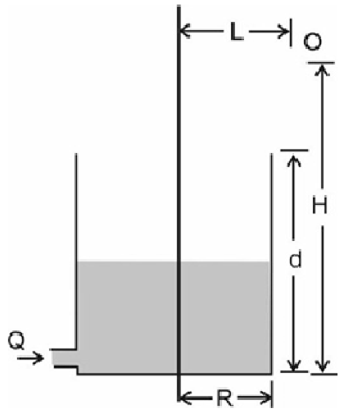
$$\Rightarrow N^2 - 160N + 1600 > 0$$

$$\Rightarrow (N-10.7)(N-148.3) > 0$$

$$\Rightarrow N < 10.7 \text{ or } N > 148.3$$

When we are putting coins one by one then we can put maximum 10 coins over the lead. For system to be in stable equilibrium.

[Q.70] A small coin is fixed at the centre of the base of an empty cylindrical steel container having radius R = 1 m and height d = 4. At time t = 0, the container starts getting filled with water at a flowrate of Q = 0.1 m^3/s without disturbing the coin. Find the approximate time when the coin will first be seen by the observer 'O' from the height of H = 5.75 m above and L = 1.5 m radially away from the coin as shown in the figure. Refractive index of water is n = 1.33



[A] 0 s

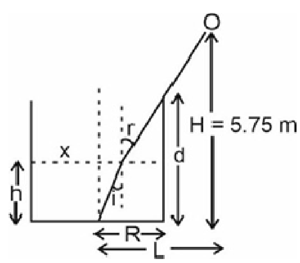
[B] 32 s

[C] 63 s

[D] 150 s

[ANS] **C**

[SOL]



$R = 1$

$L = 1.5m$

$d = 4m$

$$\tan r = \frac{L-R}{H-d} = \frac{2}{7} = \frac{x}{d-h}$$

$$\tan i = \frac{R-x}{h}$$

Using Snell's law

$$\mu \sin i = \sin r \Rightarrow \tan i = \frac{3}{\sqrt{203}}$$

$x = (d - h)\tan r = R - h\tan i$

$\Rightarrow h = \dfrac{d\tan r - R}{\tan r - \tan i} = 1.9\,m$

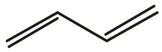
Volume $= \pi r^2 h = 3.14 \times 1.9$

$t = \dfrac{3.14 \times 1.9}{0.1} = 60\,s$

Most appropriate answer is 63 s.

PART-II : CHEMISTRY

[Q.71] A hydrocarbon X with molecular formula C_4H_6 decolorizes bromine water and forms a white precipitate in ethanolic $AgNO_3$ solution. Treatment of X with $HgCl_2$ in aqueous H_2SO_4 produces a compound, which gives a yellow precipitate when treated with I_2 and NaOH. The structure of X is :

[A] 

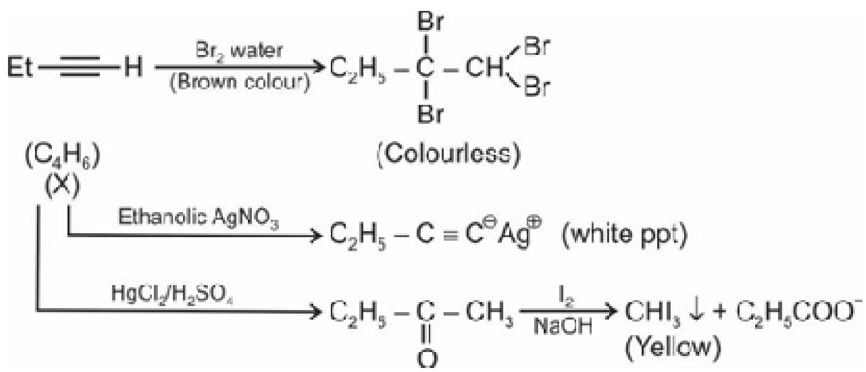
[B] 

[C] 

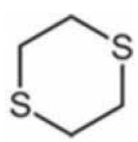
[D] 

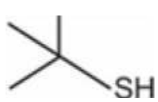
[ANS] **D**

[SOL]

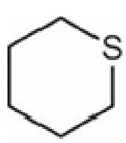


[Q.72] 0.102 g of an organic compound X was oxidized with fuming nitric acid. The resulting solution, after reaction with an excess of aqueous $BaCl_2$, produced 0.223 g of $BaSO_4$ as a precipitate. Compound X is likely to be [Given : Atomic wt. of Ba = 137]

[A] 

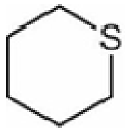
[B] 

[C] 

[D] 

[ANS] **D**

[SOL] % of S in compound $= \frac{32 \times 0.233}{233 \times 0.102} \times 100 = 31.37\,\%$

% of S in  $= \frac{32}{102} \times 100 = 31.37\,\%$

[Q.73] The specific heat of a certain substance is 0.86 J g^{-1} K^{-1}. Assuming ideal solution behavior, the energy required (in J) to heat 10 g of 1 molal of its aqueous solution from 300 K to 310 K is closest to [Given: Molar mass of the substance = 58 g mol^{-1}; specific heat of water = 1.2 J g^{-1} K^{-1}]

[A] 401.7

[B] 424.7

[C] 420.0

[D] 86.0

[ANS] **A**

[SOL] ∵ 1000 g solution has 1 mol (or 58 g) substance

∵ 10 g solution will have 0.58 g substance

1 g solution will have (10 – 0.58)g water = 9.42 g water

So, the energy required = (0.58 × 0.86 × 10) + (9.42 × 4.2 × 10)

= 400.628

= 401.7 (in J)

[Q.74] Strength of a H_2O_2 solution is labeled as 1.79 N. Its strength can also be expressed as closest to

[A] 20 volume

[B] 5 volume

[C] 10 volume

[D] 15 volume

[ANS] **C**

[SOL] ∵ Volume strength (H_2O_2) = 11.2 × molarity

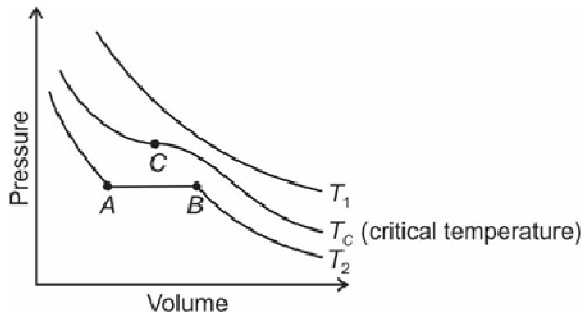
$= 5.6 \times \text{Normality}$

∵ Volume strength (H_2O_2) = 5.6 × 1.79

= 10.024

$\simeq 10\,\text{volume}$

[Q.75] The isotherms of a gas are shown below :



Among the following,

(i) At T_1, the gas cannot be liquefied

(ii) At point B, liquid starts to appear at T_2

(iii) T_c is the highest temperature at which the gas can be liquified

(iv) At point A, a small increase in pressure condenses the whole system to a liquid

The correct statements are :

[A] Only (i) and (ii)

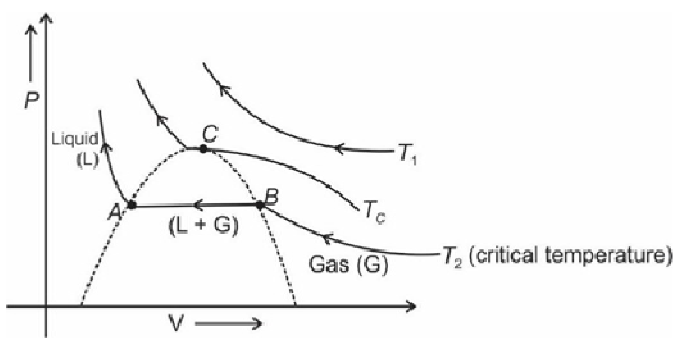
[B] Only (i), (iii) and (iv)

[C] Only (ii), (iii) and (iv)

[D] (i), (ii), (iii) and (iv)

[ANS] **D**

[SOL]



Above critical temperature (T_C), gas cannot liquify even applying high pressure.

At point B, liquid starts to appear at T_2. While at point A, a small increase in pressure condenses the whole system to a liquid.

PART-II : BIOLOGY

[Q.76] Anthropocene refers to the geological age during which

[A] The earliest hominids radiated from their ancestral forms

[B] Human activity significantly influenced climate and environment

[C] Arthropod radiation was highest

[D] Arthropod radiation significantly influenced climate and environment

[ANS] **B**

[SOL] 'Anthropo' implies 'man' and 'cene' for 'new'. Anthrocene epoch is an unofficial unit of geological time used to describe the most recent period in Earth's history when human activity started to have an significant effect on our planet's climate and ecosystems.

[Q.77] Match the vitamins listed in **column I** with the diseases caused due to their deficiency in **column II.**

	Column I		**Column II**
P.	Vitamin A	(i)	Pellegra
Q.	Vitamin B_2	(ii)	Rickets
R.	Vitamin D	(iii)	Ariboflavinosis
S.	Vitamin B_{12}	(iv)	Night blindness
		(v)	Pernicious anaemia

Choose the CORRECT combination

[A] P(iv); Q(ii); R(iii); S(v)

[B] P(i); Q(ii); R(iv); S(iii)

[C] P(iv); Q(iii); R(ii); S(v)

[D] P(iii); Q(iv); R(v); S(i)

[ANS] **C**

[SOL] Deficiency of Vitamin A causes night blindness because vitamin A is responsible for synthesis of visual pigment. Vitamin B_2 is also called riboflavin. Ariboflavinosis is a deficiency disease due to inadequate intake of riboflavin and characterised by sores on the mouth. Deficiency of vitamin D causes rickets in childhood and osteomalacia in adulthood. Deficiency of vitamin B_{12} causes pernicious anaemia.

[Q.78] An adult mammal with 50 kg body weight has the following functional parameters of its lungs.

Inspiratory reserve volume = 40 ml/kg body weight

Expiratory reserve volume = 15 ml/kg body weight

Vital capacity = 60 ml/kg body weight

Breathing rate = 20/min

The volume (in litre) of air that its lungs displaces in 24 hours is

[A] 72,000

[B] 7,200

[C] 3,600

[D] 1,200

[ANS] **B**

[SOL] Vital capacity (VC) = IRV + ERV + TV

TV = VC – (IRV + ERV)

= 60 mL/k – (40 mL/kg + 15 mL/kg)

= 5 mL/kg

TV of an adult mammal of 50 kg body weight = 5 mL/kg × 50 kg

= 250 mL

TV/min = 250 mL × 20 = 5000 mL

Volume of air that lungs displace in 24 hours (TV/day) = TV/min × 60 × 24

= 5000 mL × 60 × 24

= 7,200, 000 mL

= 7,200 L.

[Q.79] In a breed of dog, long–haired phenotype is recessive to short–hair. In a littler one pup is short haired and its sibling is long–haired. Consider the following possible phenotypes of the parents.

i. Both parents are short–haired

ii. Both parents are long–haired

iii. One parent is short–haired and one is long–haired

Choose the CORRECT combination of the possible parental phenotypes.

[A] i only

[B] ii only

[C] iii only

[D] i or iii

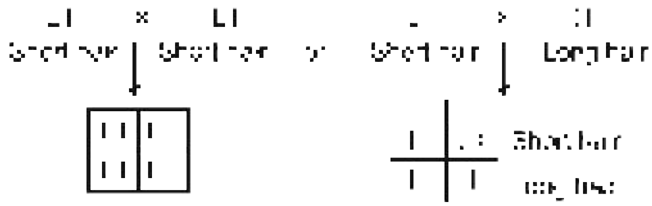
[ANS] **D**

[SOL] Long haired dog = l l

Short haired dog = L L

One pup is short haired that can be = L L or L l and its sibling is long haired = l l

So parent can be either



SO both (i) and (iii) are possible.

[Q.80] In medical diagnostics for a disease, sensitivity (denoted by a) of a test refers to the probability that a test result is positive for a person with the disease, whereas specificity (denoted by b) refers to the probability that a person without disease tests negative. A diagnostic test for COVID–19 has the values of a = 0.99 and b = 0.99. If the prevalence of COVID–19 in a population is estimated to be 10%, what is the probability that a randomly chosen parson tests positive for COVID–19?

[A] 0.099

[B] 0.10

[C] 0.108

[D] 0.11

[ANS] **C**

[SOL] P(PR) = Probability of getting positive report

P(PA) = Probability of actual COVID-19 positive

$P\left(\overline{P}_A\right)$ = Probability of not actually COVID-19 positive

$$P(P_R) = P(P_R / P_A).P(P_A) + P\left(P_R / \overline{P}_A\right).P\left(\overline{P}_A\right)$$

$$= \left(\frac{99}{100} \times \frac{10}{100}\right) + \left(\frac{1}{100} \times \frac{99}{100}\right)$$

$$= \frac{990 + 90}{10000} = \frac{108}{1000} = 0.108$$

$$P(P_R) = 0.108.$$