# JEE (ADVANCED) 2024 PAPER-1 

## [PAPER WITH SOLUTION]

## HELD ON SUNDAY $26^{\text {TH }}$ MAY 2024

## PHYSICS

## SECTION 1 (Maximum Marks :12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 ln all other cases.
[:Q.1] A dimensionless quantity is constructed in terms of electronic charge $e$, permittivity of free space $\varepsilon_{0}$, Planck's constant $h$, and speed of light $c$. If the dimensionless quantity is written as $e^{\alpha} \varepsilon_{0}{ }^{\beta} h^{\gamma} C^{\delta}$ and $n$ is a non-zero integer, then ( $\alpha, \beta, \gamma, \delta$ ) is given by
[:A] $\quad(2 n-n,-n,-n)$
[:B] $\quad(n,-n,-2 n,-n)$
[:C] $\quad(n,-n,-n,-2 n)$
[:D] $\quad(2 n,-n,-2 n,-2 n)$
[:ANS] A
[:SOLN] $\left[\mathrm{e}^{\alpha} \varepsilon_{0}^{\beta} \mathrm{h}^{\gamma} \mathrm{c}^{\delta}\right]$
$M^{0} L^{0} T^{0} A^{0}=[A T]^{\alpha}\left[M^{-1} L^{-3} T^{4} A^{2}\right]^{\beta}\left[\mathrm{ML}^{2} \mathrm{~T}^{-1}\right]^{\gamma}\left[\mathrm{LT}^{-1}\right]^{\delta}$
$\Rightarrow \quad-\beta+\gamma=0 \quad \ldots$ (i) $\quad \Rightarrow \beta=\gamma$
and $\alpha+2 \beta=0$
and $\quad-3 \beta+2 \gamma+\delta=0$
and $\quad \alpha+4 \beta-\gamma-\delta=0$
After solving (i), (ii), (iii) and (iv)
We get, $\alpha=2 n, \beta=-n, \gamma=-n, \delta=-n$
[:Q.2] An infinitely long wire, located on the $z$-axis, carries a current I along the $+z$-direction and produces the magnetic field $\vec{B}$. The magnitude of the line integral $\int \overrightarrow{\mathrm{B}} \cdot \overrightarrow{\mathrm{d} l}$ along a straight line from the point $(-\sqrt{3} a, a, 0)$ to $(a, a, 0)$ is given by
[ $\mu_{0}$ is the magnetic permeability of free space.]
[:A] $7 \mu_{0} / / 24$
[:B] $7 \mu_{0} / / 12$
[:C] $\quad \mu_{0} / / 8$
[:D] $\quad \mu_{0} / / 6$
[:ANS] A
[:SOLN]

$\sin \theta=\frac{a}{\sqrt{a^{2}+x^{2}}}$
$\int \vec{B} \cdot d \vec{\ell}=\int \frac{-\mu_{0} I}{2 \pi \sqrt{\mathrm{a}^{2}+\mathrm{x}^{2}}} \sin \theta \mathrm{dx}$
$\int_{-\sqrt{3} a}^{a} \frac{-\mu_{0} l a}{2 \pi\left(a^{2}+x^{2}\right)} d x$
$\left.=\frac{\mu_{0} l a}{2 \pi} \frac{1}{a} \tan ^{-1}\left(\frac{x}{a}\right)\right]_{-\sqrt{3} a}^{a}$
$=\frac{\mu_{0} \mathrm{I}}{2 \pi}\left[\tan ^{-1}(1)-\tan ^{-1}(\sqrt{3})\right]$
$=\frac{-\mu_{0} \mathrm{l}}{2 \pi}\left(\frac{\pi}{4}+\frac{\pi}{3}\right)=\frac{-7 \mu_{0} \mathrm{l}}{24}$, so the magnutide is $\frac{7 \mu_{0} \mathrm{l}}{24}$
[:Q.3] Two beads, each with charge $q$ and mass $m$, are on a horizontal, frictionless, non- conducting, circular hoop of radius R. One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by
[ $\varepsilon_{0}$ is the permittivity of free space.]
[:A] $\quad q^{2} /\left(4 \pi \varepsilon_{0} R^{3} m\right)$
[:B] $\quad q^{2} /\left(32 \pi \varepsilon_{0} R^{3} m\right)$
[:C] $\quad q^{2} /\left(8 \pi \varepsilon_{0} R^{3} m\right)$
$[: D] \quad q^{2} /\left(16 \pi \varepsilon_{0} R^{3} m\right)$
[:ANS] B
[:SOLN]

$F_{R}=-\frac{k Q^{2}}{4 R^{2}} \sin \theta$
$F_{R}=\frac{-k Q^{2}}{4 R^{2}} \theta=m a$
$x=R 2 \theta \quad(x \rightarrow$ small displacement $)$
so, $F_{R}=\frac{-k Q^{2} x}{8 R^{3}}=m a$
$a=\frac{-k Q^{2} x}{8 m R^{3}}=-w^{2} x$
so $w^{2}=\frac{k Q^{2}}{8 \mathrm{mR}^{3}}=\frac{\mathrm{Q}^{2}}{32 \pi \varepsilon_{0} \mathrm{mR}^{3}}$
[:Q.4] A block of mass 5 kg moves along the $x$-direction subject to the force $F=(-20 x+10) \mathrm{N}$, with the value of $x$ in metre. At time $t=0 \mathrm{~s}$, it is at rest at position $\mathrm{x}=1 \mathrm{~m}$. The position and momentum of the block at $t=(\pi / 4)$ s are
[:A] $\quad-0.5 \mathrm{~m}, 5 \mathrm{~kg} \mathrm{~m} / \mathrm{s}$
[:B] $\quad 0.5 \mathrm{~m}, 0 \mathrm{~kg} \mathrm{~m} / \mathrm{s}$
[:C] $\quad 0.5 \mathrm{~m},-5 \mathrm{~kg} \mathrm{~m} / \mathrm{s}$
[:D] $\quad-1 \mathrm{~m}, 5 \mathrm{~kg} \mathrm{~m} / \mathrm{s}$
[:ANS] C
[:SOLN] $F=-20 x+10$

$$
\begin{aligned}
& a=\frac{f}{m}=-4 x+2 \quad\left\{\begin{array}{l}
\text { given } \\
u=0, \text { at } t=0 \text { and } x=1 m
\end{array}\right. \\
& \frac{v d v}{d x}=-4 x+2 \\
& \int_{0}^{v} v d v=\int_{1}^{x}(-4 x+2) d x \\
& \frac{v^{2}}{2}=\left[-2 x^{2}+2 x\right]_{1}^{x}=-2 x^{2}+2 x-(-2+2) \\
& \Rightarrow v^{2}=-4 x^{2}+4 x \Rightarrow v=2 \sqrt{x=x^{2}}
\end{aligned}
$$

So, $v=\frac{d x}{d t}=2 \sqrt{x-x^{2}}$
$\int_{1}^{\mathrm{x}} \frac{\mathrm{dx}}{\sqrt{\mathrm{x}-\mathrm{x}^{2}}}=\int_{0}^{\pi / 4} 2 \mathrm{dt}$
$\left.\Rightarrow \sin ^{-1}(2 x-1)\right|_{1} ^{x}=2(\pi / 4)=\pi / 2$
$\Rightarrow \sin ^{-1}(2 x-1)-\sin ^{-1}(1)=\frac{\pi}{2}$
$\sin ^{-1}(2 x-1)=\pi$
$2 x-1=0$
$X=0.5 \mathrm{~m}$
$x=0.5$

v will be negative
$v=-2 \sqrt{0.5-(0.5)^{2}}=-1 \mathrm{~m} / \mathrm{s}$
$\mathrm{So} / \mathrm{P}=\mathrm{mv}=-5 \mathrm{~kg} \mathrm{~m} / \mathrm{s}$

## SECTION 2 (Maximum Marks :12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is A correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks: -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correctanswers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing $\operatorname{ONLY}(A)$ will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get -2 marks.
[:Q.5] A particle of mass m is moving in a circular orbit under the influence of the central force $F(r)=-k r$, corresponding to the potential energy $V(r)=k r^{2} / 2$, where $k$ is a positive force constant and $r$ is the radial distance from the origin. According to the Bohr's quantization rule, the angular momentum of the particle is given by $L=n \hbar$, where $\hbar=h /(2 \pi)$, $h$ is the Planck's constant, and $n$ a positive integer. If $v$ and $E$ are the speed and total energy of the particle, respectively, then which of the following expression(s) is (are) correct?
[:A] $\quad r^{2}=n \hbar \sqrt{\frac{1}{m k}}$
[:B] $\quad v^{2}=\hbar n \sqrt{\frac{k}{m^{3}}}$
[:C] $\frac{\mathrm{L}}{\mathrm{mr}^{2}}=\sqrt{\frac{\mathrm{k}}{\mathrm{m}}}$
[:D] $\quad E=\frac{n \hbar}{2} \sqrt{\frac{k}{m}}$
[:ANS] A, B, C
[:SOLN] $f(r)=-k r$
$v(r)=\frac{k r^{2}}{2}$
$\mathrm{L}=\mathrm{L}=\frac{\mathrm{nh}}{2 \pi}=\mathrm{mvr} \quad \ldots \ldots$ (i) $\Rightarrow \mathrm{v}=\left(\frac{\mathrm{n} \hbar}{2 \pi \mathrm{mr}}\right)=\frac{\mathrm{n} \hbar}{\mathrm{mr}}$
So, $|f(r)|=\frac{m v^{2}}{r}$ for circular motion
$\Rightarrow \mathrm{kr}=\frac{\mathrm{mv}}{} \mathrm{r}^{2}$
$v^{2}=\frac{k v^{2}}{m}$
$\ldots .$. (ii) $\Rightarrow \frac{v}{r}=\sqrt{\frac{k}{m}}$
$\left(\frac{\mathrm{n} \hbar}{\mathrm{mr}}\right)^{2}=\frac{\mathrm{kr}}{}{ }^{2} \mathrm{~m} \Rightarrow \mathrm{r}^{4}=\frac{(\mathrm{n} \hbar)^{2}}{\mathrm{mk}} \Rightarrow \mathrm{r}^{2}=\mathrm{n} \hbar \sqrt{\frac{1}{\mathrm{mk}}} \quad$ option (A)
$v^{2}=\frac{k}{m} r^{2}=\frac{k}{m} n \hbar \sqrt{\frac{1}{m k}}=n \hbar \sqrt{\frac{k}{m^{3}}}$
$\frac{L}{m r^{2}}=\frac{m v r}{m r^{2}}=\frac{v}{r}=\sqrt{\frac{k}{m}}$
$E=\frac{1}{2} m v^{2}+v(r)=\frac{1}{2} m\left[n \hbar \sqrt{\frac{k}{m^{3}}}\right]+\frac{k r^{2}}{2}$
$=\frac{\mathrm{n} \hbar}{2} \sqrt{\frac{\mathrm{k}}{\mathrm{m}}}+\frac{\mathrm{k}}{2} \quad \mathrm{n} \hbar \sqrt{\frac{1}{\mathrm{mk}}}=\mathrm{n} \hbar \sqrt{\frac{\mathrm{k}}{\mathrm{m}}} \quad \quad$ option (D)
[:Q.6] Two uniform strings of mass per unit length $\mu$ and $4 \mu$, and length $L$ and $2 L$, respectively, are joined at point $O$, and tied at two fixed ends $P$ and $Q$, as shown in the figure. The strings are under a uniform tension $T$. If we define the frequency $v_{0}=\frac{1}{2 L} \sqrt{\frac{T}{\mu}}$, which of the following statement(s) is (are) correct?

[:A] With a node at O , the minimum frequency of vibration of the composite string is $\mathrm{v}_{0}$.
[:B] With an antinode at O , the minimum frequency of vibration of the composite string is $2 v_{0}$.
[:C] When the composite string vibrates at the minimum frequency with a node at $O$, it has 6 nodes, including the end nodes.
[:D] No vibrational mode with an antinode at O is possible for the composite string.
[:ANS] A, C, D
[:SOLN]

$\mathrm{v}_{0}=\frac{1}{2 \mathrm{~L}} \sqrt{\frac{\mathrm{~T}}{\mu}} \& \mathrm{~T}_{2}$ tension $=\mathrm{T}$
Option (A) for min. frequency . for node at O
Suppose ' $n$ ' loops in left string and ' $m$ ' loops in right string
$\Rightarrow \frac{\mathrm{n} \lambda_{1}}{2}=\mathrm{L} \& \frac{\mathrm{~m} \lambda_{2}}{2}=2 \mathrm{~L}$
$\frac{n v_{1}}{2 f \min }=L \ldots(I) \& \frac{m v_{2}}{2 f \min }=2 L$
(I) $\div$ (II)
$\frac{1}{2}=\frac{\mathrm{n}}{\mathrm{m}} \frac{\sqrt{\mathrm{t} / \mu}}{\sqrt{\mathrm{t} / 4 \mu}}=\frac{1}{4}=\frac{\mathrm{n}}{\mathrm{m}}$
So $\min ^{m} n=1$ and $m=4$
That mean $f_{\text {min }}=\frac{1}{2 L} \sqrt{\frac{T}{\mu}}=v_{0}$


Option(B) for anti - node at O'
$\frac{1}{4}=\frac{(2 n+1)}{(2 n+1)} ; n o$ any $n$ and $m$ will be possible,
So, correct (A)(C)(D).
[:Q.7] A glass beaker has a solid, plano-convex base of refractive index 1.60, as shown in the figure. The radius of curvature of the convex surface (SPU) is 9 cm , while the planar surface (STU) acts as a mirror. This beaker is filled with a liquid of refractive index $n$ up to the level QPR. If
the image of a point object O at a height of h (OT in the figure) is formed onto itself, then, which of the following option(s) is (are) correct?

[:A] For $\mathrm{n}=1.42, \mathrm{~h}=50 \mathrm{~cm}$.
[:B] For $\mathrm{n}=1.35, \mathrm{~h}=36 \mathrm{~cm}$.
[:C] For $n=1.45, h=65 \mathrm{~cm}$.
[:D] For $\mathrm{n}=1.48, \mathrm{~h}=85 \mathrm{~cm}$.
[:ANS] A, B
[:SOLN]


Object \& Image coincide that mean ('o' will be R.O.C)
the effective focal length of the system will $\left(\frac{h}{2}\right)$
$\frac{1}{\text { feff }}=\frac{2}{f_{1}}+\frac{2}{f_{2}}+\frac{1}{f_{m}}$
$\frac{1}{\text { feff }}=\frac{2(n-1)}{\infty}+\frac{2(1.6-n)}{R}+\frac{1}{\infty}$
Feff $=\frac{h}{2}=\frac{R}{2(1.6-n)} \Rightarrow h=\frac{9}{1 \cdot 6-n}$
For $\mathrm{n}=1.42, \mathrm{~h}=50 \mathrm{~cm}$
\& $n=1.35, h=36 \mathrm{~cm}$

Option (A) and (B).

## SECTION 3 (Maximum Marks :24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;
Zero Marks : 0 In all other cases.
[:Q.8] The specific heat capacity of a substance is temperature dependent and is given by the formula $\mathrm{C}=\mathrm{kT}$, where k is a constant of suitable dimensions in SI units, and T is the absolute temperature. If the heat required to raise the temperature of 1 kg of the substance from $-73^{\circ} \mathrm{C}$ to $27^{\circ} \mathrm{C}$ is nk , the value of n is $\qquad$ . [Given : $0 \mathrm{~K}=-273^{\circ} \mathrm{C}$ ]
[:ANS] 25000
[:SOLN] C = KT = Specific heat $\rightarrow \mathrm{m}=1 \mathrm{~kg}$
$\Delta \mathrm{Q}=\int \mathrm{msdT}=\mathrm{m} \int_{200 \mathrm{k}}^{300 \mathrm{k}} \mathrm{kTdT} \quad \begin{aligned} & -73^{\circ} \mathrm{C}=200 \mathrm{k} \\ & 27^{\circ} \mathrm{C}=300 \mathrm{k}\end{aligned}$
$\Delta Q=1 \times \frac{k}{2}\left[T^{2}\right]_{200}^{300}=\frac{k}{2}[9-4] \times 10^{4}=2.5 \times 10^{4} \times k$
$\Delta \mathrm{Q}=2500 \mathrm{k}, \mathrm{n}=25000$
[:Q.9] A disc of mass $M$ and radius $R$ is free to rotate about its vertical axis as shown in the figure. $A$ battery operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the same mass $R$ and radius $R / 2$ is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed $\omega$. If the angular speed at which the large disc rotates is $\omega / n$, then the value of $n$ is $\qquad$ .

[:ANS] 12
[SOLN] $\quad T_{\text {sys } i}=T_{\text {sys } f}=0$ about centre of big disc

$$
\begin{aligned}
& {\left[\frac{M \frac{R^{2}}{4}}{2}\right] \omega-M\left[\omega^{\prime} R\right] R-\frac{M R^{2}}{2} \omega^{\prime}=0} \\
& \frac{M R^{2}}{8} \omega=3 \frac{M R^{2}}{2} \omega^{\prime} \\
& \Rightarrow \quad \omega^{\prime}=\frac{\omega}{12} \\
& n=12
\end{aligned}
$$

[:Q.10] A point source $S$ emits unpolarized light uniformly in all directions. At two points $A$ and $B$, the ratio $r=I_{A} / I_{B}$ of the intensities of light is 2 . If a set of two polaroids having $45^{\circ}$ angle between their pass-axes is placed just before point $B$, then the new value of $r$ will be $\qquad$ .
[:ANS] 8
[SOLN] Case (1)
$r=\frac{I_{A}}{I_{B}}=2$
Case (2)
$\mathrm{r}^{\prime}=\frac{\mathrm{I}_{\mathrm{A}^{\prime}}}{\mathrm{I}_{\mathrm{B}^{\prime}}}=\frac{\mathrm{I}_{\mathrm{A}}}{\frac{\mathrm{I}_{\mathrm{B}}}{2} \cos ^{2} 45^{\circ}}=\frac{\mathrm{I}_{\mathrm{A}}}{\frac{\mathrm{I}_{\mathrm{B}}}{2} \times \frac{1}{2}}$

$=4 \frac{\mathrm{I}_{\mathrm{A}}}{\mathrm{I}_{\mathrm{B}}}=4 \times 2=8$
[:Q.11] A source (S) of sound has frequency 240 Hz . When the observer (O) and the source move towards each other at a speed $v$ with respect to the ground (as shown in Case 1 in the figure), the observer measures the frequency of the sound to be 288 Hz . However, when the observer and the source move away from each other at the same speed $v$ with respect to the ground (as shown in Case 2 in the figure), the observer measures the frequency of sound to be n Hz . The value of $n$ is $\qquad$ .

Case 1


Case 2

[:ANS] 200
[SOLN] Case (1)
$288=240\left[\frac{C+V}{C-V}\right]$
$\frac{C+V}{C-V}=\frac{288}{240}$
Case (2)
$\mathrm{f}_{\mathrm{AP}}=\mathrm{f}_{\mathrm{AC}}\left[\frac{\mathrm{C}-\mathrm{V}}{\mathrm{C}+\mathrm{V}}\right]$
$\mathrm{n}=240 \times \frac{240}{288}=200$
[:Q.12] Two large, identical water tanks, 1 and 2, kept on the top of a building of height H , are filled with water up to height $h$ in each tank. Both the tanks contain an identical hole of small radius on their sides, close to their bottom. A pipe of the same internal radius as that of the hole is connected to tank 2, and the pipe ends at the ground level. When the water flows from the tanks 1 and 2 through the holes, the times taken to empty the tanks are $t_{1}$ and $t_{2}$, respectively. If $H=\left(\frac{16}{9}\right) h$, then the ratio $t_{1} / t_{2}$ is $\qquad$ .
[:ANS] 3
[:SOLN] Case (1)

$$
\left[\frac{\mathrm{dv}}{\mathrm{dt}}\right]_{\mathrm{hole}}=\left[\frac{\mathrm{dv}}{\mathrm{dt}}\right]_{\text {tank }}
$$


$A_{0} \sqrt{2 g y}=A \times\left(-\frac{d y}{d t}\right)$
$t_{1}=-\frac{A}{A_{0}} \frac{1}{\sqrt{2 g}} \int_{y=h}^{y=0} \frac{d y}{\sqrt{y}}=\frac{A}{A_{0}} \sqrt{\frac{2 h}{g}}$
Case (2)
$\left[\frac{\mathrm{dv}}{\mathrm{dt}}\right]_{\text {hole }}=\left[\frac{\mathrm{dv}}{\mathrm{dt}}\right]_{\text {tomk }}$

$A_{o} \sqrt{2 g\left(y+\frac{16 h}{9}\right)}=A \times\left(-\frac{d y}{d t}\right)$
$t_{2}=-\frac{A}{A_{0} \sqrt{2 g}} \int_{y=h}^{y=0} \frac{d y}{\sqrt{y+\frac{16 h}{9}}}$
$=-\frac{A}{A_{0}} \frac{1}{\sqrt{2 g}} 2\left[\sqrt{y+\frac{16 h}{9}}\right]_{y=h}^{y=0}$
$=\frac{1}{3} \frac{\mathrm{~A}}{\mathrm{~A}_{0}} \sqrt{\frac{2 \mathrm{~h}}{\mathrm{~g}}}$
Then $\frac{t_{1}}{t_{2}}=\frac{1}{(1 / 3)}=3$
[:Q.13] A thin uniform rod of length $L$ and certain mass is kept on a frictionless horizontal table with a massless string of length $L$ fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point $O$. If a horizontal impulse $P$ is imparted to the rod at a distance $x$ $=\mathrm{L} / \mathrm{n}$ from the mid-point of the rod (see figure), then the rod and string revolve together around the point O , with the rod remaining aligned with the string. In such a case, the value of $n$ is $\qquad$ .

[:ANS] 18
[SOLN]

M.I. of rod about centre (O) $=\frac{m L^{2}}{12}+m\left(L+\frac{L}{2}\right)^{2}$
$I_{0}=\frac{7 m L^{2}}{3}$
Since rod is in pure rotation about ' O '
So, angular impulse $=P\left(x+\frac{3 L}{2}\right)=I_{0} \omega_{0}$
and linear impulse $=m v_{c}$
where $V_{c}=\frac{3 L}{2} \omega_{0}$
$(20 \mathrm{~m}(\mathrm{i})) m\left(\frac{3 L}{2} \omega_{0}\right)\left(x+\frac{3 L}{2}\right)=\frac{7}{3} m L^{2} \cdot \omega_{0}$
$x+\frac{3 L}{2}=\frac{14}{9} L$
$x=\frac{L}{18}$

## SECTION 4 (Maximum Marks :12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme: Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases
[:Q.14] One mole of a monatomic ideal gas undergoes the cyclic process $J \rightarrow K \rightarrow L \rightarrow M \rightarrow J$, as shown in the $\mathrm{P}-\mathrm{T}$ diagram.


Match the quantities mentioned in List-I with their values in List-II and choose the correct option. [ R is the gas constant.]

## List-I

(P) Work done in the complete cyclic process
(Q) Change in the internal energy of the gas in the process JK
(R) Heat given to the gas in the process KL
(S) Change in the internal energy of the gas in the
process MJ

## List-II

(1) $R T_{0}-4 R T_{0} \ln 2$
(2) 0
(3) $3 R T_{0}$
(4) $-2 \mathrm{RT}_{0} \ln 2$
(5) $-3 R T_{0} \ln 2$

Codes:
[:A] $\quad P \rightarrow 1 ; Q \rightarrow 3 ; R \rightarrow 5 ; S \rightarrow 4$
[:B] $\quad \mathrm{P} \rightarrow 4 ; \mathrm{Q} \rightarrow 3 ; \mathrm{R} \rightarrow 5 ; \mathrm{S} \rightarrow 2$
[:C] $\mathrm{P} \rightarrow 4 ; \mathrm{Q} \rightarrow 1 ; \mathrm{R} \rightarrow 2 ; \mathrm{S} \rightarrow 2$

$$
\text { [:D] } \quad P \rightarrow 2 ; Q \rightarrow 5 ; R \rightarrow 3 ; S \rightarrow 4
$$

[ANS.] B
[SOLN]

(P) $\quad \mathrm{W}_{\text {cyclic }}=\underset{\text { Isobaric }}{\mathrm{W}_{\mathrm{JK}}}+\underset{\text { Isothermal }}{\mathrm{W}_{\mathrm{RC}}}+\underset{\text { Isobaric }}{\mathrm{W}_{\mathrm{LM}}}+\underset{\text { Isothermal }}{\mathrm{W}_{\mathrm{MJ}}}$

$$
\begin{aligned}
& =1 \times R \times\left(3 T_{0}-T_{0}\right)+1 \times R \times 3 T_{0} \ln \frac{P_{0}}{2 P_{0}}+1 \times R \cdot\left[T_{0}-3 T_{0}\right]+1 \times R \times T_{0} \ln \frac{2 p_{0}}{P_{0}} \\
& =2 R T_{0}-3 R T_{0} \ell n 2-2 R T_{0}+R T_{0} \ln 2 \\
& =-2 R T_{0} \ell n 2
\end{aligned}
$$

(Q) $\quad \Delta \mathrm{U}_{\mathrm{JK}}=\Delta \mathrm{U}=n \mathrm{c}_{\mathrm{V}} \cdot \Delta \mathrm{T}$

$$
=1 \times \frac{3 R}{2} \times\left[3 T_{0}-T_{0}\right]=3 R T_{0}
$$

(R) $\quad \Delta \mathrm{Q}_{\mathrm{KL}}=\Delta \mathrm{Q}_{\text {isothernual }}=\Delta \mathrm{U}+\mathrm{W}=-3 \mathrm{R}_{0} \ln 2[\because \Delta \mathrm{U}=0]$
(S) $\quad \Delta \mathrm{U}_{\mathrm{MJ}}=0(\because$ Process is isothermal $)$
[:Q.15] Four identical thin, square metal sheets, $S_{1}, S_{2}, S_{3}$ and $S_{4}$, each of side a are kept parallel to each other with equal distance $\mathrm{d}(\ll \mathrm{a})$ between them, as shown in the figure Let $\mathrm{C}_{0}=\varepsilon_{0} \mathrm{a}^{2} / \mathrm{d}$, where $\varepsilon_{0}$ is the permittivity of free space


Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

## List - I

(P) The capacitance between $S_{1}$ and $S_{4}$, with $S_{2}$ and $S_{3}$ not connected, is
(Q) The capacitance between $S_{1}$ and $S_{4}$, with $S_{2}$ shorted to $S_{3}$, is
(R) The capacitance between $S_{1}$ and $S_{3}$, with
$S_{2}$ shorted to $S_{4}$, is
(S) The capacitance between $\mathrm{S}_{1}$ and $\mathrm{S}_{2}$ with
$S_{3}$ shorted to $S_{1}$, and $S_{2}$ shorted to $S_{4}$, is

List - II
(1) $3 C_{0}$
(2) $\mathrm{C}_{0} / 2$
(3) $\mathrm{C}_{0} / 3$
(4) $2 \mathrm{C}_{0} / 3$
(5) $2 \mathrm{C}_{0}$

## Codes :

[:A] $\mathrm{P} \rightarrow 3 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 4 ; \mathrm{S} \rightarrow 5$
$[: B] \quad P \rightarrow 2 ; Q \rightarrow 3 ; R \rightarrow 2 ; S \rightarrow 1$
[:C] $\quad \mathrm{P} \rightarrow 3 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 4 ; \mathrm{S} \rightarrow 1$
[:D] $\mathrm{P} \rightarrow 3 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 2 ; \mathrm{S} \rightarrow 5$
[ANS] C
[SOLN]



[:Q.16] A light ray is incident on the surface of a sphere of refractive index $n$ at an angle of incidence $\theta_{0}$.The ray partially refracts into the sphere with angle of refraction $\phi_{0}$ and then partly reflects from the back surface. The reflected ray then emerges out of the sphere after a partial refraction. The total angle of deviation of the emergent ray with respect to the incident ray is $\alpha$. Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

## List - I

(P) If $\mathrm{n}=2$ and $\alpha=180^{\circ}$, then all the possible values of $\theta_{0}$ will be
(Q) If $\mathrm{n}=\sqrt{3}$ and $\alpha=180^{\circ}$, then all the possible values of $\theta_{0}$ will be
(R) If $\mathrm{n}=\sqrt{3}$ and $\alpha=180^{\circ}$, then all the possible values of $\phi_{0}$ will be
(S) If $\mathrm{n}=\sqrt{2}$ and $\theta_{0}=45^{\circ}$, then all the possible values of $\alpha$ will be

## List - II

(1) $30^{\circ}$ and $0^{\circ}$
(2) $60^{\circ}$ and $0^{\circ}$
(3) $45^{\circ}$ and $0^{\circ}$
(4) $150^{\circ}$
(5) $0^{\circ}$

## Codes :

[:A] $\mathrm{P} \rightarrow 5 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 1 ; \mathrm{S} \rightarrow 4$
[:B] $\quad \mathrm{P} \rightarrow 5 ; \mathrm{Q} \rightarrow 1 ; \mathrm{R} \rightarrow 2 ; \mathrm{S} \rightarrow 4$
[:C] $\mathrm{P} \rightarrow 3 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 1 ; \mathrm{S} \rightarrow 4$
[:D] $\quad \mathrm{P} \rightarrow 3 ; \mathrm{Q} \rightarrow 1 ; \mathrm{R} \rightarrow 2 ; \mathrm{S} \rightarrow 5$
[:ANS] A
[:SOLN]

$\alpha=\left(\theta_{0}-\phi_{0}\right)+\left(\pi-2 \phi_{0}\right)+\left(\theta_{0}-\phi_{0}\right)$
$\alpha=\pi+2 \theta_{0}-4 \phi_{0}$
$\sin \theta_{0}=n \sin \phi_{0}$
For (P)
$n=2, \alpha=180$
if $\alpha=\pi, 2 \theta_{0}-4 \phi_{0}=0$
$\theta_{0}=2 \phi_{0}$
$\sin \theta_{0}=2 \sin \left(\frac{\theta_{0}}{2}\right)$
$P \rightarrow(5)$
For $Q, n=\sqrt{3}, \alpha=180$
$\theta_{0}=2 \phi_{0}$
$Q \rightarrow(2)$
$\sin \theta_{0}=\sqrt{3} \sin \left(\frac{\theta_{0}}{2}\right)$
$\theta_{0}=60,0^{\circ}$
[:Q.17] The circuit shown in the figure contains an inductor $L$, a capacitor $C_{0}$, a resistor $R_{0}$ and an ideal battery. The circuit also contains two keys $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$. Initially, both the keys are open and there is no charge on the capacitor. At an instant, key $\mathrm{K}_{1}$ is closed and immediately after
this the current in $R_{0}$ is found to be $I_{1}$. After a long time, the current attains a steady state value $I_{2}$. Thereafter, $\mathrm{K}_{2}$ is closed and simultaneously $\mathrm{K}_{1}$ is opened and the voltage across $\mathrm{C}_{0}$ oscillates with amplitude $\mathrm{V}_{0}$ and angular frequency $\omega_{0}$.


Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

## List-I

(P) The value of $\mathrm{I}_{1}$ in Ampere is
(Q) The value of $\mathrm{I}_{2}$ in Ampere is
(R) The value of $\omega_{0}$ in kilo-radians/s is
(S) The value of $\mathrm{V}_{0}$ in Volt is
(4) 20
(5) 200

## Codes:

[:A] $\mathrm{P} \rightarrow 1 ; \mathrm{Q} \rightarrow 3 ; \mathrm{R} \rightarrow 2 ; \mathrm{S} \rightarrow 5$
[:B] $\mathrm{P} \rightarrow 1 ; \mathrm{Q} \rightarrow 2 ; \mathrm{R} \rightarrow 3 ; \mathrm{S} \rightarrow 5$
[:C] $P \rightarrow 1 ; Q \rightarrow 3 ; R \rightarrow 2 ; S \rightarrow 4$
[:D] $\quad \mathrm{P} \rightarrow 2 ; \mathrm{Q} \rightarrow 5 ; \mathrm{R} \rightarrow 3 ; \mathrm{S} \rightarrow 4$
[:ANS] A
[:SOLN] (P)


At $=\mathrm{t}=0$
Inductor behave as infinite resistance

$$
l_{1}=0
$$

(Q) After long time, inductor behaves as zero resistance,

$$
I_{\max }=\frac{20}{5}=4 \mathrm{~A}
$$

(R)


When $\mathrm{K}_{1}$ is open and $\mathrm{K}_{2}$ is closed
The circuit will be LC - oscillation with angular frequency.

$$
\begin{aligned}
& \mathrm{w}_{0}=\frac{1}{\sqrt{\text { LC }}}=\frac{1}{\sqrt{25 \times 10^{-3} \times 10 \times 10^{-6}}} \\
& \begin{aligned}
=\frac{1}{\sqrt{25 \times 10^{-8}}} & =\frac{10^{4}}{5} \mathrm{rad} / \mathrm{s}=2000 \mathrm{rad} / \mathrm{s} \\
& =2 \text { kilo-rad } / \mathrm{s}
\end{aligned}
\end{aligned}
$$

(S) $\quad \frac{1}{2} \mathrm{LI}_{0}^{2}=\frac{1}{2} \mathrm{CV}_{0}^{2}$

$$
V_{0}=\sqrt{L / C} \quad I_{0}=\sqrt{\frac{25 \times 10^{-3}}{10 \times 10^{-6}}} \times 4=200 \mathrm{Volt} .
$$

