JEE (ADVANCED) 2024 PAPER-2

[PAPER WITH SOLUTION]

HELD ON SUNDAY 26TH MAY 2024

MATHEMATICS

SECTION-1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.

[:Q.1] Considering only the principal values of the inverse trigonometric functions, the value of

$$\tan\left(\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$$ is

[:A] $\frac{7}{24}$

[:B] $\frac{-7}{24}$

[:C] $\frac{-5}{24}$

[:D] $\frac{5}{24}$

[:ANS] **B**

[:SOLN] $\tan\left(\tan^{-1}\left(\frac{3}{4}\right)-\tan^{-1}\left(\frac{4}{3}\right)\right)$

$$=\frac{\frac{3}{4}-\frac{4}{3}}{1+1}=-\frac{7}{24}$$

[:Q.2] Let $S=\left\{(x,y)\in\mathbb{R}\times\mathbb{R}: x\geq 0, y\geq 0, y^2\leq 4x, y^2\leq 12-2x, \text{ and } 3y+\sqrt{8}x\leq 5\sqrt{8}\right\}$. If the area of the region S is $\alpha\sqrt{2}$, then α is equal to

[:A] $\frac{17}{2}$

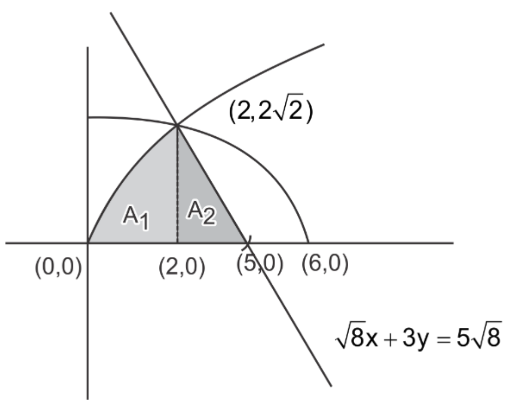
[:B] $\frac{17}{3}$

[:C] $\frac{17}{4}$

[:D] $\frac{17}{5}$

[:ANS] **B**

[:SOLN]



$$A_1=\frac{2}{3}\times 2\times 2\sqrt{2}=\frac{8\sqrt{2}}{3}$$

$$A_2=\frac{1}{2}\times 3\times 2\sqrt{2}=3\sqrt{2}$$

Total Area = $A_1+A_2=\alpha\sqrt{2}$

$$\frac{8\sqrt{2}}{3}+3\sqrt{2}=\alpha\sqrt{2}$$

$\alpha = \frac{8}{3} + 3 = \frac{17}{3}$

[:Q.3] Let $k \in \mathbb{R}$. If $\lim_{x \to 0+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$, then the value of k is

[:A] 1

[:B] 2

[:C] 3

[:D] 4

[:ANS] **B**

[:SOLN] $\lim_{x \to 0^+} (\sin(\sin kx) + \cos x + x)^{2/x} = e^6$

$\Rightarrow \lim_{x \to 0^+} \left(\frac{\sin(\sin kx) + \cos x + x - 1}{x} \right) 2 = 6$

$\Rightarrow \lim_{x \to 0^+} (k \cos(\sin kx).\cos kx - \sin x + 1) = 3$

$\Rightarrow k + 1 = 3 \qquad \Rightarrow k = 2$

[:Q.4] Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$

Then which of the following statements is TRUE?

[:A] f(x) = 0 has infinitely many solutions in the interval $\left[\frac{1}{10^{10}}, \infty\right)$.

[:B] f (x) = 0 has no solutions in the interval $\left[\frac{1}{\pi}, \infty\right)$.

[:C] The set of solutions of f (x) = 0 in the interval $\left(0, \frac{1}{10^{10}}\right)$ is finite.

[:D] f (x) = 0 has more than 25 solutions in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$.

[:ANS] **D**

[:SOLN] $f : R \to R$

$$f(x) = x^2 \sin\left(\frac{\pi}{x^2}\right) = 0,\ x \neq 0$$

$$\sin\left(\frac{\pi}{x^2}\right) = 0 \Rightarrow \frac{\pi}{x^2} = n$$

$$x^2 = \frac{1}{n}$$

$$x = \pm\frac{1}{\sqrt{n}},\ n \in N$$

(A) $x = \frac{1}{\sqrt{n}} = \left[\frac{1}{10^{10}}, \infty\right) \Rightarrow \frac{1}{\sqrt{n}} \geq \frac{1}{100}$

$\Rightarrow \sqrt{n} \leq 10^{10}$

$1 \leq n \leq 10^{20}, n \in N.$

finite value of n.

(B) $x = \frac{1}{\sqrt{n}} = \left[\frac{1}{\pi}, \infty\right) \Rightarrow \frac{1}{\sqrt{n}} \geq \frac{1}{\pi}$

$\sqrt{n} \leq \pi$

$1 \leq n \leq \pi^2$

n = {1,2,3, …10}

(C) $x = \left(0, \frac{1}{10^n}\right) = \frac{1}{\sqrt{n}} \Rightarrow 0 < \frac{1}{\sqrt{n}} < \frac{1}{10^{10}}.$

$n > 10^{20} \Rightarrow$ Infinite value.

(D) $x = \left(\frac{1}{\pi^2}, \frac{1}{\pi}\right) = \frac{1}{\sqrt{n}}$

$\frac{1}{\pi^2} < \frac{1}{\sqrt{n}} < \frac{1}{\pi} \Rightarrow \frac{1}{\pi^4} < \frac{1}{n} < \frac{1}{\pi^2} \Rightarrow \pi^2 < n < \pi^4 \quad \Rightarrow 9.86 < n < 97.4$

$n = \{10, 11, 12, ...97\}$

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get −2 marks

[:Q.5] Let S be the set of all $(\alpha,\beta)\in\mathbb{R}\times\mathbb{R}$ such that $\lim\limits_{x\to\infty}\dfrac{\sin(x^2)(\log_e x)^\alpha \sin\left(\dfrac{1}{x^2}\right)}{x^{\alpha\beta}(\log_e(1+x))^\beta}=0.$

Then which of the following is (are) correct?

[:A] $(-1,3)\in S$

[:B] $(-1,1)\in S$

[:C] $(1,-1)\in S$

[:D] $(1,-2)\in S$

[:ANS] **B,C**

[:SOLN] $\lim\limits_{x\to\infty} \dfrac{\sin(x^2)(\ln x)^{\alpha}\sin\left(\dfrac{1}{x^2}\right)}{x^{\alpha}\beta(\ln(1+x))^{\beta}} = 0$

$\Rightarrow \lim\limits_{x\to\infty} \left(\dfrac{\sin\frac{1}{x^2}}{\frac{1}{x^2}}\right).(\sin x^2).\dfrac{(\ln x)^{\alpha}}{x^{(\alpha\beta+2)}(\ln(1+x))^{\beta}} = 0$

1 finite = [-1, 1]

$\Rightarrow \lim\limits_{x\to\infty} \dfrac{(\ln x)^{\alpha}}{x^{(\alpha\beta+2)}(\ln(1+x))^{\beta}} = 0$

(A) $(\alpha,\beta) = (-1,3) \Rightarrow L = \lim\limits_{x\to\infty} \dfrac{x}{(\ln\ x)(\ln(1+x))^3} \approx \infty$

(B) $(\alpha,\beta) = (-1,1) \Rightarrow L = \lim\limits_{x\to\infty} \dfrac{1}{x\ln x\ln(1+x)} = 0$

(C) $(\alpha,\beta) = (1,-1) \Rightarrow L = \lim\limits_{x\to\infty} \dfrac{\ln x\ln(1+x)}{x} = 0$

(D) $(\alpha,\beta) = (1,-2) \Rightarrow L = \lim\limits_{x\to\infty} \ln x\ln(1+x) \approx \infty$

[:Q.6] A straight line drawn from the point P (1, 3, 2), parallel to the line $\dfrac{x-2}{1} = \dfrac{y-4}{2} = \dfrac{z-6}{1}$, intersects the plane L_1 : x – y + 3z = 6 at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane L_2 : 2x – y + z = -4 at the point R. Then which of the following statements is (are) TRUE?

[:A] The length of the line segment PQ is $\sqrt{6}$

[:B] The coordinates of R are (1, 6, 3)

[:C] The centroid of the triangle PQR is $\left(\dfrac{4}{3},\dfrac{14}{3},\dfrac{5}{3}\right)$

[:D] The perimeter of the triangle PQR is $\sqrt{2}+\sqrt{6}+\sqrt{11}$

[:ANS] **A,C**

[:SOLN] # P(1,3,2)

$Q(1+\lambda, 3+2\lambda, 2+\lambda)$ lies on the plane x - y + 3z =6

$\Rightarrow 1+\lambda-3-2\lambda+6+3\lambda = 6$

$\Rightarrow \lambda = 1$

$\Rightarrow Q(2,5,3)$

$R(2+\lambda, 5-\lambda, 3+3\lambda)$ lies on 2x - y + z = -4

$\Rightarrow 4+2\lambda-5+\lambda+3+3\lambda=-4$

$\Rightarrow \lambda=-1$

$\Rightarrow R(1,6,0)$

PQ = $\sqrt{6}$

$PQ+QR+RP=\sqrt{6}+\sqrt{11}+\sqrt{13}$

centroid of ΔPQR = $\left(\frac{4}{3},\frac{14}{3},\frac{5}{3}\right)$

[:Q.7] Let A_1, B_1, C_1 be three points in the x-y-plane. Suppose that the lines A_1C_1 and B_1C_1 are tangents to the curve $y^2 = 8x$ at A_1 and B_1, respectively. If O = (0,0) and C_1 = (-4,0), then which of the following statements is (are) TRUE?

[:A] The length of the line segment OA_1 is $4\sqrt{3}$

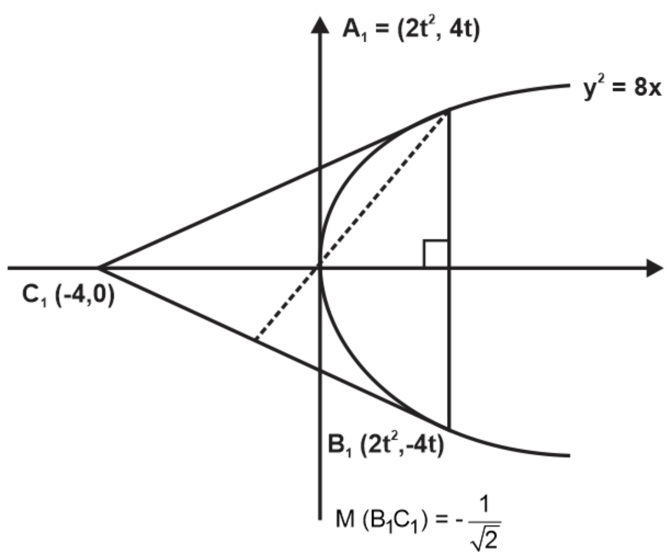
[:B] The length of the line segment $A_1 B_1$ is 16

[:C] The orthocenter of the triangle $A_1 B_1 C_1$ is (0, 0)

[:D] The orthocenter of the triangle $A_1 B_1 C_1$ is (1, 0)

[:ANS] **A,C**

[:SOLN]



Tangent at A(t) : ty = (x + 2t²) Passes (-4, 0)

$\Rightarrow -4+2t^2=0 \Rightarrow t^2=2.$

$A_1(4, 4\sqrt{2}), B_1(4, -4\sqrt{2})$

$OA_1 = OB_1 = 4\sqrt{3}$

$A_1B_1 = 8\sqrt{2}$

Orthocentre of $\Delta A_1B_1C_1 : H(x, 0)$:

Eq^n of $A_1H : y - 4\sqrt{2} = \sqrt{2}(x - 4)$

$y = 0 \Rightarrow x = 0 \Rightarrow H = (0, 0)$

SECTION-3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

[:Q.8] Let $f : R \to R$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in R$, and $g : R \to (0, \infty)$ be a function such that If $g(x+y) = g(x)g(y)$ for all $x, y \in R$. If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0)$ is ________.

[:ANS] **51**

[:SOLN] Let $f(x) = kx$, as $f\left(-\frac{3}{5}\right) = 12$

$\therefore \frac{-3k}{5} = 12 \qquad \therefore k = -20$

Let, $g(x) = a^x$, given that $g\left(-\frac{1}{3}\right) = 2$

$a^{\frac{-1}{3}} = 2 \Rightarrow a = \frac{1}{8} \qquad \therefore f(x) = -20x,\ g(x) = \left(\frac{1}{8}\right)^x$

From Question : $\left\{\frac{-20}{4}+\left(\frac{1}{8}\right)^{-2}-8\right\}\times\left(\frac{1}{8}\right)^{0}=51$

[:Q.9] A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For i = 1,2,3, let W_i, G_i, and B_i denote the events that the ball drawn in the i^{th} draw is a white ball, green ball, and blue ball, respectively. If the probability $P\left(W_1\cap G_2\cap B_3\right)=\frac{2}{5N}$ and the conditional probability $P\left(B_3\left|W_1\cap G_2\right.\right)=\frac{2}{9}$, then N equals __________.

[:ANS] **11**

[:SOLN] $P(B_3\mid W_1\cap G_2)=\frac{2}{9}$

$\Rightarrow \frac{P(B_3\cap W_1\cap G_2)}{P(W_1\cap G_2)}=\frac{2}{9}$

$\Rightarrow \frac{\frac{2}{5N}}{\frac{3}{N}\times\frac{6}{N-1}}=\frac{2}{9}\left(\because P(W_1\cap G_2\cap B_3)=\frac{2}{5N}\right)$

$\Rightarrow \frac{(N-1)}{3\times3\times5}=\frac{2}{9}$

$N=11$

[:Q.10] Let the function $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=\frac{\sin x}{e^{\pi x}}\frac{\left(x^{2023}+2024x+2025\right)}{\left(x^2-x+3\right)}+\frac{2}{e^{\pi x}}\frac{\left(x^{2023}+2024x+2025\right)}{\left(x^2-x+3\right)}.$$

Then the number of solutions of f(x) = 0 in $\mathbb{R}$ is __________.

[:ANS] **1**

[:SOLN] $f(x)=0$

$\Rightarrow \frac{\sin x+2}{e^{\pi x}(x^2-x+3)}.(x^{2023}+2024x+2025)=0$

$\Rightarrow x^{2023}+2024x+2025=0$ $(\therefore \sin x+2>0$ and $e^{\pi x}(x^2-x+3)>0\,\forall x\in R)$

Let $g(x)=x^{2023}+2024x+2025$

$\therefore g'(x) = 2023x^{2022} + 2024 > 0 \forall x \in R.$

$\therefore$ g(x) is strictly increase polynomial of odd degree.

$\therefore g(x) = 0$ has only one real solution.

[:Q.11] Let $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real numbers α, β and γ, we have $15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha\left(2\vec{p} + \vec{q}\right) + \beta\left(\vec{p} - 2\vec{q}\right) + \gamma\left(\vec{p} \times \vec{q}\right)$, then the value of γ is ________.

[:ANS] **2**

[:SOLN] $\left(15\hat{i} + 10\hat{j} + 6k\right).\left(\vec{p} \times \vec{q}\right) = 0 + 0 + \gamma\left(\vec{P} \times \vec{q}\right)^2$

$$L.H.S = \begin{vmatrix} 15 & 10 & 6 \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix} = 60 + 10 - 18 = 52$$

$$R.H.S = \gamma \begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix}^2 = \gamma(16 + 1 + 9) = \gamma.26$$

So $\gamma.26 = 52$

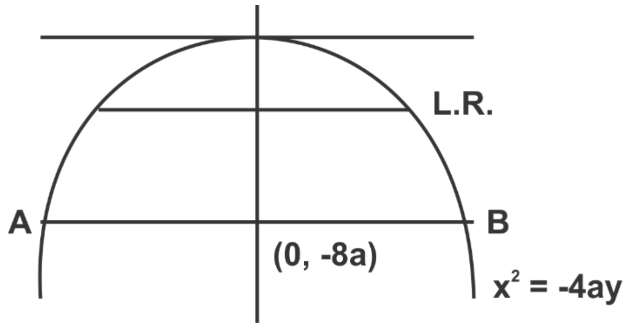
[:Q.12] A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0, -\alpha)$ to the parabola $x^2 = -4ay$, where $a > 0$. Let L be the line passing through $(0, -\alpha)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB. If r : s = 1 : 16, then the value of 24a is __________.

[:ANS] **12**

[:SOLN] Equation of normal to $x^2 = -4ay$ is : $x + yt + 2at + at^3 = 0$

Passing through $(0, -\alpha) \Rightarrow -\alpha t + 2at + at^3 = 0 \Rightarrow \alpha = 2a + at^2$

Slope of normal $= \frac{-1}{t} = \frac{1}{\sqrt{6}} \quad \therefore t = -\sqrt{6} \quad \therefore \alpha = 8a$



For A & B, $y = -8a \quad \therefore x = \pm 4\sqrt{2}a$

$\therefore |AB| = 8\sqrt{2}a,\ |LR| = 4a$

given $\dfrac{|LR|}{|AB|^2} = \dfrac{4a}{128a^2} = \dfrac{1}{16} \Rightarrow a = \dfrac{1}{2} \quad \therefore 24a = 12$

[:Q.13] Let the function $f:[1,\infty) \to \mathbb{R}$ be defined by

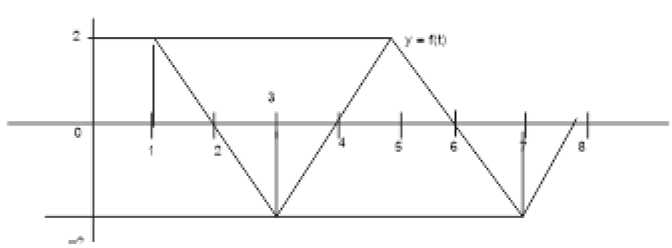
$$f(t) = \begin{cases} (-1)^{n+1}2, & \text{if } t = 2n-1,\ n \in \mathbb{N}, \\ \dfrac{(2n+1-t)}{2} f(2n-1) + \dfrac{(t-(2n-1))}{2} f(2n+1), & \text{if } 2n-1 < t < 2n+1,\ n \in \mathbb{N}. \end{cases}$$

Define $g(x) = \int_1^x f(t)\,dt,\ x \in (1,\infty).$ Let α denote the of solutions of the equation g(x) = 0 in the interval (1,8] and $\beta = \lim_{x \to 1+} \dfrac{g(x)}{x-1}.$ Then the value of $\alpha + \beta$ is equal to ________.

[:ANS] **5**

[:SOLN] $f(t) = \begin{cases} (-1)^{n+1}.2, t = 2n-1, n \in N. \\ \dfrac{2n+1-t}{2}.(-1)^{n+1}.2 + \dfrac{k-(2n-1)}{2}.(-1)^{n+2}.2, 2n-1 < t < 2n+1, n \in N \end{cases}$

$= \begin{cases} (-1)^{n+1}.2, t = 2n-1, n \in N \\ (-1)^{n+1}(4n-2t), 2n-1 < t < 2n+1, n \in N \end{cases}$



Clearly, $g(x)=\int_{1}^{x} f(t)dt=0$

$\Rightarrow x=3,5 \text{ or } 7$

$\Rightarrow \alpha=3$

$\beta=\lim_{x\to1^+}\frac{g(x)}{x-1}=\lim_{x\to1^+}\frac{g'(x)}{1}=g'(1)$ (L'hopital's Rule)

$=f(x)=2$

$\therefore \alpha+\beta=5$

SECTION-4 (Maximum Marks : 12)

- This section contains **TWO (02)** questions.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct numerical value is entered in the designated place;

Zero Marks : 0 In all other cases.

PARAGRAPH "I"

Let S = {1,2,3,4,5,6} and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a,b)\in R$, we have $|a-b|\geq 2$.

Let $Y=\{R\in X:\text{ The range of R has exactly one element}\}$ and

$Z=\{R\in X: R \text{ is a function from S to S}\}$. Let n(A) denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

[:Q.14] If n(X) = $^{m}C_{6}$, then the value of m is ________.

PARAGRAPH "I"

Let S = {1,2,3,4,5,6} and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a,b) \in R$, we have $|a-b| \geq 2$.

Let $Y = \{R \in X: \text{ The range of R has exactly one element}\}$ and

$Z = \{R \in X : R \text{ is a function from S to S}\}$. Let n(A) denote the number of elements in a set A.

(There are two questions based on PARAGRAPH "I", the question given below is one of them)

[:ANS] **20**

[:SOLN]

a	b
1	3,4,5,6
2	4,5,6
3	1,5,6
4	1,2,6
5	1,2,3
6	1,2,3,4

So total 20 pairs satisfy the condition $|a-b| \geq 2$,

Out of which R can have any 6 pairs.

So no. of possible relations R = $^{20}C_6$

$\therefore n(x) = {}^{20}C_6 = {}^{20}C_6$

$\Rightarrow m = 20$

[:Q.15] If the value of n(Y) + n(Z) is k^2, then |k| is _____________.

[:ANS] **36**

[:SOLN] $\because$ each b is related to only 3 or 4 elements in S, so the range of R cannot have exactly one element.

$\therefore$ n(y) = 0

For R to be a function from S to S, each element in S must be related to one and only one element in S.

So n(z) = 4 x 3 x 3 x 3 x 3 x 4

(as 1 & 6 have 4 options; 2, 3, 4, 5 have 3 options each)

$\therefore$ n(y) + n(z) = 0 + 36^2 = k^2

$\Rightarrow |k| = 36$

PARAGRAPH "II"

Let $f:\left[0,\frac{\pi}{2}\right]\to[0,1]$ be the function defined by f(x) = $\sin^2 x$ and let $g:\left[0,\frac{\pi}{2}\right]\to[0,\infty)$ be the function defined by $g(x)=\sqrt{\frac{\pi x}{2}-x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

[:Q.16] The value of $2\int_0^{\frac{\pi}{2}} f(x)g(x)dx-\int_0^{\frac{\pi}{2}} g(x)dx$ is ________.

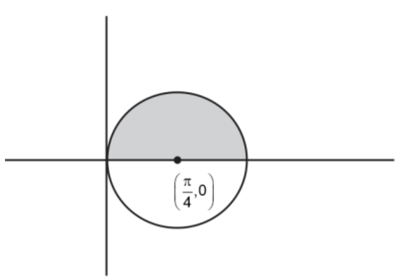
[:ANS] **0**

[:SOLN] $I=\int_0^{\pi/2} f(x).g(x)dx$

$I=\int_0^{\pi/2} \sin^2 x\times\sqrt{\frac{\pi x}{2}-x^2}dx$, Applying P-IV

$I=\int_0^{\pi/2} \cos^2 x\times\sqrt{\frac{\pi x}{2}-x^2}dx \quad \left[\text{as } g\left(\frac{\pi}{2}-x\right)=g(x)\right]$

$2I=\int_0^{\pi/2} \sqrt{\frac{\pi x}{2}-x^2}dx$



$\text{Area}=\pi.\frac{r^2}{2}$

$$=\pi.\frac{\left(\frac{\pi}{4}\right)^2}{2}=\frac{\pi^3}{32}$$

$$2I=\int_0^{\pi/2} g(x)dx=0$$

PARAGRAPH "II"

Let $f:\left[0,\frac{\pi}{2}\right]\rightarrow[0,1]$ be the function defined by f(x) = $\sin^2 x$ and let $g:\left[0,\frac{\pi}{2}\right]\rightarrow[0,\infty)$ be the function defined by $g(x)=\sqrt{\frac{\pi x}{2}-x^2}$.

(There are two questions based on PARAGRAPH "II", the question given below is one of them)

[:Q.17] The value of $\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}} f(x)g(x)dx$ is ___________.

[:ANS] **0.25**

[:SOLN] $\frac{16}{\pi^3}\times\frac{\pi^3}{64}=\frac{1}{4}=0.25$