JEE (ADVANCED) 2024 PAPER-1

[PAPER WITH SOLUTION]

HELD ON SUNDAY 26TH MAY 2024

MATHEMATICS

SECTION 1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

[:Q.1] Let f(x) be a continuously differentiable function on the interval $(0, \infty)$ such that f(1) = 2 and

$$\lim_{t \to x} \frac{t^{10} f(x) - x^{10} f(t)}{t^9 - x^9} = 1$$

for each x > 0. Then, for all x > 0, f(x) is equal to

[A] $\frac{31}{11x} - \frac{9}{11}x^{10}$

[B] $\frac{9}{11x} + \frac{13}{11}x^{10}$

[C] $\frac{-9}{11x} + \frac{31}{11}x^{10}$

[D] $\frac{13}{11x} + \frac{9}{11}x^{10}$

[:ANS] **B**

[:SOLN] $$\lim_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$$

$$\Rightarrow \lim_{t\to x}\frac{10t^9f(x)-x^{10}f(t)}{9t^8}=1 \qquad \text{(L' hopital's Rule)}$$

$$\Rightarrow \frac{10x^9f(x)-x^{10}f^1(x)}{9x^8}=1$$

$$\Rightarrow \frac{10x^9f(x)-x^{10}f^1(x)}{x^{20}}=\frac{9x^8}{x^{20}}$$

$$\Rightarrow \frac{d}{dx}\frac{f(x)}{x^{10}}=\frac{-9}{x^{12}}$$

$$\Rightarrow \frac{f(x)}{x^{10}}=\frac{9x^{-11}}{11}+C$$

$$\Rightarrow f(x)=\frac{9}{11x}+Cx^{10}$$

$$f(1)=2\Rightarrow C=2-\frac{9}{11}=\frac{13}{11}$$

$$\therefore f(x)=\frac{9}{11x}+\frac{13x^{10}}{11}$$

[:Q.2] A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is

[A] $\frac{1}{12}$

[B] $\frac{1}{7}$

[C] $\frac{5}{7}$

[D] $\frac{5}{12}$

[:ANS] **C**

[:SOLN] E_1 : Student knows the answer

E_2 : Student guessed the answer

A **:** Student answers correctly

Let $P(E_1) = \alpha$

$\Rightarrow$ $P(E_2) = 1 - \alpha$

Given $P\left(\frac{A}{E_2}\right) = \frac{1}{2}$

Now, $P\left(\frac{E_2}{A}\right) = \frac{P(E_2).P\left(\frac{A}{E_2}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)}$

$$\frac{1}{6} = \frac{(1-\alpha)\times\frac{1}{2}}{\alpha + (1-\alpha).\frac{1}{2}}$$

$$\frac{\alpha+1}{2} = 3 - 3\alpha \Rightarrow \frac{7\alpha}{2} = \frac{5}{2}$$

$$\Rightarrow \alpha = \frac{5}{7}$$

[:Q.3] Let $\frac{\pi}{2} < x < \pi$ be such that $\cot x = \frac{-5}{\sqrt{11}}$. Then

$$\left(\sin\frac{11x}{2}\right)(\sin 6x - \cos 6x) + \left(\cos\frac{11x}{2}\right)(\sin 6x + \cos 6x)$$

is equal to

[A] $\frac{\sqrt{11}-1}{2\sqrt{3}}$

[B] $\dfrac{\sqrt{11}+1}{2\sqrt{3}}$

[C] $\dfrac{\sqrt{11}+1}{3\sqrt{2}}$

[D] $\dfrac{\sqrt{11}-1}{3\sqrt{2}}$

[:ANS] **B**

[:SOLN] $\sin\dfrac{11x}{2}(\sin 6x-\cos 6x)+\cos\dfrac{11x}{2}(\sin 6x+\cos 6x)$

$$=\left(\sin\frac{11x}{2}\sin 6x+\cos\frac{11x}{2}\cos 6x\right)+\left(\sin 6x\cos\frac{11x}{2}-\cos 6x\sin\frac{11x}{2}\right)$$

$$=\cos\left(6x-\frac{11x}{2}\right)+\sin\left(6x-\frac{11x}{2}\right)$$

$$=\cos\frac{x}{2}+\sin\frac{x}{2}$$

$$=\sqrt{\frac{1+\cos x}{2}}+\sqrt{\frac{1-\cos x}{2}}\qquad\left(\because \frac{x}{2}\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)\right)$$

$$=\sqrt{\frac{1+\frac{5}{\sqrt{11+25}}}{2}}+\sqrt{\frac{1-\frac{5}{\sqrt{11+25}}}{2}}$$

$$=\frac{\sqrt{11}+1}{2\sqrt{3}}$$

[:Q.4] Consider the ellipse $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$. Let S (p, q) be a point in the first quadrant such that $\dfrac{p^2}{9}+\dfrac{q^2}{4}>1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the center of the ellipse. If the area of the triangle ΔORT is $\dfrac{3}{2}$, then which of the following options is correct?

[A] $q=2, p=3\sqrt{3}$

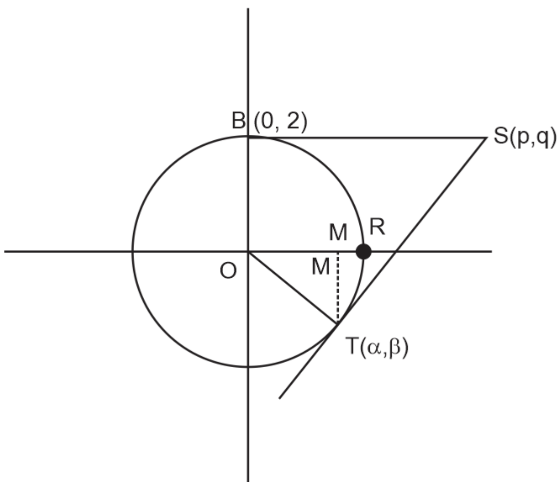
[B] $q=2, p=4\sqrt{3}$

[C] $q=1, p=5\sqrt{3}$

[D] $q=1, p=6\sqrt{3}$

[:ANS] **A**

[:SOLN]



$$ar(ORT)=\frac{1}{2}\times OR\times MT$$

$$\Rightarrow \frac{3}{2}=\frac{1}{2}\times 3\times|\beta|$$

$$\Rightarrow \beta=-1 \qquad (\because \beta<0)$$

Now, $\frac{\alpha^2}{9}+\frac{(-1)^2}{4}=1$

$$\Rightarrow \alpha=\frac{3\sqrt{3}}{2}$$

$\therefore$ tangent at $T\left(\frac{3\sqrt{3}}{2},-1\right)$ is

$$\frac{x\frac{3\sqrt{3}}{2}}{9}+\frac{y(-1)}{4}=1$$

Putting $y = 2, \frac{x}{2\sqrt{3}} - \frac{1}{2} = 1 \Rightarrow x = 3\sqrt{3} = p$

$\therefore (p, q) = (3\sqrt{3}, 2)$

SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	ONLY if (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;

choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;

choosing ONLY (B) and (D) will get +2 marks;

choosing ONLY (A) will get +1 mark;

choosing ONLY (B) will get +1 mark;

choosing ONLY (D) will get +1 mark;

choosing no option (i.e. the question is unanswered) will get 0 marks; and

choosing any other combination of options will get −2 marks

[:Q.5] Let $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$, $T_1 = \left\{\left(-1+\sqrt{2}\right)^n : n \in \mathbb{N}\right\}$, and $T_2 = \left\{\left(1+\sqrt{2}\right)^n : n \in \mathbb{N}\right\}$.

Then which of the following statements is (are) TRUE?

[A] $\mathbb{Z} \cup T_1 \cup T_2 \subset S$

[B] $T_1 \cap \left(0, \frac{1}{2024}\right) = \phi$, where ϕ denotes the empty set.

[C] $T_2 \cap (2024, \infty) \neq \phi$

[D] For any given $a, b \in \mathbb{Z}$, $\cos\left(\pi\left(a+b\sqrt{2}\right)\right) + i\sin\left(\pi\left(a+b\sqrt{2}\right)\right) \in \mathbb{Z}$ if and only if b = 0, where $i = \sqrt{-1}$.

[:ANS] **A,C,D**

[:SOLN] (A) $Z = \{a + b\sqrt{2}; a \in Z, b = 0\} \subset S$

$\because (-1+\sqrt{2})^n = p + q\sqrt{2}$ and $(1+\sqrt{2})^n = r + s\sqrt{2}$ for some p,q,r,s, $\in z$

$\therefore T_1, T_2 \subset S.$

$\therefore Z \cup T_1 \cup T_2 \subset S$

(B) $\because 0 < -1 + \sqrt{2} < 1$

$\therefore (-1+\sqrt{2})^n \in (0, \frac{1}{2024})$ for some large $n \in N$

$\therefore T_1 \cap \left(0, \frac{1}{2024}\right) \neq \phi$

(C) $1+\sqrt{2} > 1 \Rightarrow (1+\sqrt{2})^n > 2024$ for some large $n \in N$

So $T_2 \cap (2024, \infty) \neq \phi$

(D) $\cos(\pi(a + b\sqrt{2})) + i\sin(\pi(a + b\sqrt{2})) \in Z$

$\Leftrightarrow \pi(a + b\sqrt{2}) = n\pi$ for some $n \in Z$

$\Leftrightarrow a + b\sqrt{2} = n$ for some $n \in Z$

$\Leftrightarrow b = 0$

[:Q.6] Let $\mathbb{R}^2$ denote $\mathbb{R} \times \mathbb{R}$, Let

S = {(a, b, c) : a, b, c $\in \mathbb{R}$ and $ax^2 + 2bxy + cy^2 > 0$ for all (x, y) $\in \mathbb{R}^2 - \{(0, 0)\}$}.

Then which of the following statements is (are) TRUE?

[A] $\left(2, \frac{7}{2}, 6\right) \in S$

[B] If $\left(3, b, \frac{1}{12}\right) \in S$, then $|2b| < 1$.

[C] For any given $(a, b, c) \in S$, the system of linear equations

$$ax + by = 1$$
$$bx + cy = -1$$

has a unique solution.

[D] For any given $(a, b, c) \in S$, the system of linear equations

$$(a + 1)x + by = 0$$
$$bx + (c + 1)y = 0$$

has a unique solution.

[:ANS] **B,C,D**

[:SOLN] $ax^2 + 2bxy + cy^2 > 0 \ \forall \ x, y \in R^2 - \{(0,0)\}$

$\Leftrightarrow a > 0$ and $4b^2 - 4ac < 0$

(A) $b^2 - ac = \frac{49}{4} - 12 > 0$

$\therefore$ (A) is false.

(B) $b^2 - 3 \times \frac{1}{12} < 0 \Rightarrow b^2 < \frac{1}{4} \Rightarrow |2b| < 1$

$\therefore$ (B) is true.

(C) $b^2 - ac \neq 0 \Leftrightarrow \frac{a}{b} \neq \frac{b}{c}$

$\therefore$ Unique solution

(D) $\frac{a+1}{b} \neq \frac{b}{c+1}$

$\Leftrightarrow ac + a + c + 1 \neq b^2$

$\Leftrightarrow \underbrace{(ac - b^2)}_{+ve} + \underbrace{a + c + 1}_{+ve} \neq 0$

Which is true. So unique solution.

[:Q.7] Let $\mathbb{R}^3$ denote the three-dimensional space. Take two points P = (1, 2,3) and Q = (4,2,7). Let dist(X, Y) denote the distance between two points X and Y in $\mathbb{R}^3$, Let

$$S = \left\{X \in \mathbb{R}^3 : (\text{dist}(X, P))^2 - (\text{dist}(X, Q))^2 = 50\right\} \text{ and}$$

$$T = \left\{Y \in \mathbb{R}^3 \ : (\text{dist}\,(Y,Q))^2 - (\text{dist}(Y,P))^2 = 50\right\}.$$

Then which of the following statements is (are) TRUE?

[A] There is a triangle whose area is 1 and all of whose vertices are from S.

[B] There are two distinct points L and M in T such that each point on the line segment LM is also in T.

[C] There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T

[D] There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

[:ANS] **A,B,C,D**

[:SOLN] $S \equiv \{(x, y, z) : (x-1)^2 + (y-2)^2 + (z-3)^2 - (x-4)^2 - (y-2)^2 - (z-7)^2 = 50\}$

$\Rightarrow S \equiv \{(x, y, z) : 6x + 8z = 105 \}$

Similarly, $T = \{(x, y, z) : 6x + 8z = 5\}$

$\therefore$ S & T are two parallel planes at a distance of 10 units from each other.

SECTION-3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Full Marks : 0 In all other cases.

[:Q.8] Let $a = 3\sqrt{2}$ and $b = \dfrac{1}{5^{1/6}\sqrt{6}}$. If $x, y \in \mathbb{R}$ are such that

$$3x + 2y = \log_a (18)^{\frac{5}{4}} \quad \text{and}$$
$$2x - y = \log_b \left(\sqrt{1080}\right),$$

then 4x + 5y is equal to ________.

[:ANS] **2**

[:SOLN] $3x + 2y = \log_a 18^{5/4} = \log_{3\sqrt{2}}(3\sqrt{2})^{5/2} = \dfrac{5}{2}$ … (i)

$2x - y = \log_{\frac{1}{5^{1/6}\sqrt{6}}}(5 \times 6^3) = -3$ … (ii)

Solving equation (i) and (iii)

$x = \dfrac{-1}{2}, y = 2$

$\therefore\ 4x + 5y = 8$

[:Q.9] Let f(x) =$x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that f(1) = –9. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If $\alpha_1, \alpha_2, \alpha_3$ and α_4 are all the roots of the equation f(x) = 0, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to ___.

[:ANS] **20**

[:SOLN] $f'(x) = 4x^3 + 3ax^2 + 2bx = x(4x^2 + 3ax + 2b)$

$\because\ i\sqrt{3}$ is a root of $f'(x) = 0$, so $-i\sqrt{3}$ is also a root.

$\therefore\ 4x^2 + 3ax + 2b \equiv 4(x + i\sqrt{3})(x - i\sqrt{3}) \equiv 4(x^2 + 3)$

$\therefore$ a = 0, b = 6

f(x) $= x^4 + ax^3 + bx^2 + c$

$= x^4 + 6x^2 + c$

$f(1) = -9 \Rightarrow c = -16$

$f(x) = 0 \Rightarrow\ x^4 + 6x^2 - 16 = 0 \Rightarrow x^2 = 2$ or -8

$\therefore\ |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$

= 2 + 2 + 8 + 8 = 20

[:Q.10] Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} :\ a, b, c, d, e \in \{0, 1\} \text{ and } |A| \in \{-1, 1\} \right\}$, where $|A|$ denotes the determinant of A. Then the number of elements in S is _______.

[:ANS] 16

[:SOLN] Det (A) = (d – e) + c(b – a)

Det A = $\pm$ 1

There will be 8 elements in S for c = 0 & with c = 1 also there will be 8 elements

hence, the required number = 16.

[:Q.11] A group of 9 students, s_1, s_2, ……, s_9, is to be divided to form three teams X, Y and Z and of sizes 2, 3, and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then the number of ways to form such teams, is _______.

[:ANS] **665**

[:SOLN] $\text{Required no. of ways} = \frac{9!}{2!3!4!} - \left\{\frac{8!}{1!3!4!} + \frac{8!}{2!2!4!} - \frac{7!}{1!2!4!}\right\}$

= 665

[:Q.12] Let $\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{OQ} = \hat{i} + \frac{\beta-1}{\beta}\hat{j} + \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ be three vectors, where $\alpha, \beta \in \mathbb{R} - \{0\}$ and O denotes the origin. If $\left(\overrightarrow{OP} \times \overrightarrow{OQ}\right) \cdot \overrightarrow{OR} = 0$ and the point $(\alpha, \beta, 2\}$ lies on the plane 3x + 3y – z + l = 0, then the value of l is ____.

[:ANS] **5**

[:SOLN] $\Rightarrow \begin{vmatrix} \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} \alpha-1 & \alpha & \alpha \\ \beta & \beta-1 & \beta \\ 2 & 2 & 1 \end{vmatrix} = 0 \left(R_1 \to \alpha R_1, R_2 \to \beta R_2, R_3 \to 2R_3\right)$

$\Rightarrow \begin{vmatrix} \alpha+\beta+1 & \alpha+\beta+1 & \alpha+\beta+1 \\ \beta & \beta-1 & \beta \\ 2 & 2 & 1 \end{vmatrix} = 0 \ (R_1 \to R_1 + R_2 + R_3)$

$\Rightarrow \alpha + \beta + 1 = 0$

$(\alpha, \beta, 2)$ lies on $3x + 3y - z + \ell = 0$

$\Rightarrow 3\alpha + 3\beta - 2 + \ell = 0$

$\Rightarrow \ell = 2 - 3(\alpha + \beta) = 2 + 3 = 5$

[:Q.13] Let X be a random variable, and let P (X = x) denote the probability that X takes the value x. Suppose that the points (x, P (X = x)), x = 0,1,2,3,4, lie on a fixed straight line in the xy -

plane, and P (X = x) = 0 for all $x \in \mathbb{R} - \{0,1,2,3,4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is α, then the value of 24α is ______

[:ANS] 42

[:SOLN] (0, P(0)), (1, P(1)), (2, P(2)),(3, P(3)), (4, P(4)) lie on a fixed straight line, therefore,

P(1) – P(0) = P(2) – P(1) = P(3) – P(2) = P(4) – P(3) = m ('say')

P(0) + P(1) + P(2) + P(3) + P(4) = 1

5P(0) + 10 m = 1 … (1)

Further, $\sum_{x=0}^{4} xP(x) = \frac{5}{2}$

$\Rightarrow 10P(0) + 30m = \frac{5}{2}$ … (2)

Now, $\sum_{x=0}^{4} x^2P(x) = 8$

$\therefore$ variance $\alpha = 8 - \frac{25}{4} = \frac{7}{4}$

$\Rightarrow 24\alpha = 42$

SECTION-4 (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II.**
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

[:Q.14] Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3 × 3 matrix $M = (a_{ij})_{3\times3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for i =1,2,3 and j =1,2,3.

Match each entry **in List-I** to the correct entry in **List-II.**

	List-I		List-II
(P)	The number of matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that 0 $R_i = C_j = 0$ for all i, j, is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $C_j = 0$ for all j, is	(2)	12
(R)	Let $M = (a_{ij})_{3\times3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then the number of elements in the set $\left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R},\ M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix}\right\}$ is	(3)	infinite
(S)	Let $M = (a_{ij})_{3\times3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then the absolute value of the determinant of M is	(4)	6
		(5)	0

The correct option is

[A] $(P) \to (4)$ $(Q) \to (2)$ $(R) \to (5)$ $(S) \to (1)$

[B] $(P) \to (2)$ $(Q) \to (4)$ $(R) \to (1)$ $(S) \to (5)$

[C] $(P) \to (2)$ $(Q) \to (4)$ $(R) \to (3)$ $(S) \to (5)$

[D] $(P) \to (1)$ $(Q) \to (5)$ $(R) \to (3)$ $(S) \to (4)$

[:ANS] **C**

[:SOLN] $\alpha + \beta = -1, \alpha\beta = -1$

(P) $\because 1 + \alpha + \beta = 0$

$\therefore\ R_i = C_j = 0\ \forall$ i, j

$\Leftrightarrow$ each row and each column is a permutation of $(1, \alpha, \beta)$

$\therefore$ $R_1 \rightarrow$ 13 ways

$R_2 \rightarrow$ 2 ways

$R_3 \rightarrow$ 1 way

So no. of matrices = $6 \times 2 \times 1 = 12$

(Q) $\because$ M is symmetric

$\therefore$ $C_j = 0 \Leftrightarrow R_j = 0$

So, $C_1 \rightarrow$ 6 ways

after which R_1 is also fixed and remaining only one way.

so, no. of matrices = 6

(R) Let $M = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

$$\therefore M\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{12} \\ 0 \\ -a_{23} \end{bmatrix} \Leftrightarrow \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -a \\ 0 \\ c \end{bmatrix}$$

$\therefore$ $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

So infinite solutions.

(S) $R_i = 0$ for all i

$\Leftrightarrow$ each row is permutation of $(1, \alpha, \beta)$

So adding the 3 columns we get $(1 + \alpha + \beta)$ common. So value of $|M|$ is 0.

[:Q.15] Let the straight line y = 2x touch a circle with center $(0, \alpha)$, $\alpha > 0$, and radius r at a point A_1. Let B_1 be the point on the circle such that the line segment $A_1 B_1$ is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$.

Match each entry in **List-I** to the correct entry in **List-II.**

	List-I		**List-II**
(P)	α equals	(1)	$(-2, 4)$
(Q)	*r* equals	(2)	$\sqrt{5}$
(R)	A_1 equals	(3)	$(-2, 6)$
(S)	B_1 equals	(4)	5
		(5)	(2, 4)

The correct option is

[A] $(P)\to(4)$ $(Q)\to(2)$ $(R)\to(1)$ $(S)\to(3)$

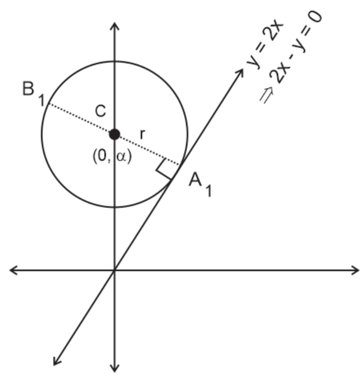
[B] $(P)\to(2)$ $(Q)\to(4)$ $(R)\to(1)$ $(S)\to(3)$

[C] $(P)\to(4)$ $(Q)\to(2)$ $(R)\to(5)$ $(S)\to(3)$

[D] $(P)\to(2)$ $(Q)\to(4)$ $(R)\to(3)$ $(S)\to(5)$

[:ANS] **C**

[:SOLN]



Distance of line from centre = r

$r = \frac{\alpha}{\sqrt{5}}$ … (i)

Given $\alpha + r = 5 + \sqrt{5}$ … (ii)

Solving (i) and (ii)

$r = \sqrt{5}, \alpha = 5$

Find foot of $\perp^r$ from C to $2x - y = 0$ to find $A_1 \to A_1\,(2, 4)$

$B_1\,(-2, 6)$

Ans. (C)

[:Q.16] Let $\gamma \in \mathbb{R}$ be such that the lines $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$ and $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma}$ intersect. Let R_1 be the point of intersection of L_1 and L_2. Let O = (0,0,0), and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2.

List-I		List-II	
(P)	γ equals	(1)	$-\hat{i} - \hat{j} + \hat{k}$

(Q)	possible choice for $\hat{n}$ is	(2)	$\sqrt{\frac{3}{2}}$
(R)	$\overrightarrow{OR_1}$ equals	(3)	1
(S)	A possible value of $\overrightarrow{OR_1} \cdot \hat{n}$ is	(4)	$\frac{1}{\sqrt{6}}\hat{i}-\frac{2}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}$
		(5)	$\sqrt{\frac{2}{3}}$

The correct option is

[A] $(P)\to(3)$ $(Q)\to(4)$ $(R)\to(1)$ $(S)\to(2)$

[B] $(P)\to(5)$ $(Q)\to(4)$ $(R)\to(1)$ $(S)\to(2)$

[C] $(P)\to(3)$ $(Q)\to(4)$ $(R)\to(1)$ $(S)\to(5)$

[D] $(P)\to(3)$ $(Q)\to(1)$ $(R)\to(4)$ $(S)\to(5)$

[:ANS] **C**

[:SOLN] Since lines intersect

$$\therefore \begin{vmatrix} (-11+16) & (-21+11) & (-29+4) \\ 1 & 2 & 3 \\ 3 & 2 & \gamma \end{vmatrix} = 0$$

$$\downarrow$$

Solving this , $\gamma = 1$

Now, $\frac{x+11}{1}=\frac{y+21}{2}=\frac{z+29}{3}=a$

$\frac{x+16}{3}=\frac{y+11}{2}=\frac{z+4}{1}=b$

To find pt of intersection

$a - 11 = 3b - 16$

$a - 3b = -5$ … (i)

$2a - 21 = 2b - 11$

$a - b = 5$ … (ii)

Solving (i) and (ii)

$a = 10,\ b = 5$

Pt. of int. of L_1 & L_2 = (a – 11, 2a – 21, 3a – 29)

R_1 (–1, –1, 1)

Normal to the plane containing L_1 and L_2

$$\hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = -4\hat{i} + 8\hat{j} - 4\hat{k}$$

$$\hat{n} = \frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$$

$$\boxed{\begin{aligned} \overrightarrow{OR_1}\cdot\hat{n} &= \frac{-1}{\sqrt{6}} + \frac{2}{\sqrt{6}} + \frac{1}{\sqrt{6}} \\ &= \frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}} \end{aligned}}$$

[:Q.17] Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be functions defined by

$$f(x) = \begin{cases} x\,|x|\sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 1-2x, & 0 \le x \le \frac{1}{2}, \\ 0, & \text{otherwise}. \end{cases}$$

Let a, b, c, d, $\in\mathbb{R}$. Define the function $h:\mathbb{R}\to\mathbb{R}$ by

$$h(x) = a\,f(x) + b\left(g(x) + g\left(\frac{1}{2} - x\right)\right) + c\left(x - g(x)\right) + d\,g(x),\ x\in\mathbb{R}.$$

Match each entry in **List-I** to the correct entry in **List-II.**

List-I		List-II	
(P)	If a = 0, b = 1, c = 0, and d = 0 then	(1)	h is one-one.
(Q)	If a = 1, b = 0, c = 0, and d = 0 then	(2)	h is onto.
(R)	If a = 0, b = 0, c = 1, and d = 0 then	(3)	h is differentiable on $\mathbb{R}$
(S)	If a = 0, b = 0, c = 0, and d = 1 then	(4)	the range of h is [0,1].
		(5)	the range of h is {0,1}.

The correct option is

[A] $(P)\to(4)\quad(Q)\to(3)\quad(R)\to(1)\quad(S)\to(2)$

[B] $(P)\to(5)\quad(Q)\to(2)\quad(R)\to(4)\quad(S)\to(3)$

[C] $(P)\to(5)\quad(Q)\to(3)\quad(R)\to(2)\quad(S)\to(4)$

[D] $(P)\to(4)\quad(Q)\to(2)\quad(R)\to(1)\quad(S)\to(3)$

[:ANS] **C**

[:SOLN] (P) a = c = d = 0, b = 1

$$\Rightarrow h(x)=g(x)+g\left(\frac{1}{2}-x\right)$$

$$=\begin{cases}1-2x+1-2\left(\frac{1}{2}-x\right), & 0\le x\le\frac{1}{2}\\ 0\ +\ 0 & ,\text{otherwise}\end{cases}$$

$$=\begin{cases}1, & 0\le x\le\frac{1}{2}\\ 0, & \text{otherwise}\end{cases}$$

range of h is {0, 1}

(Q) a = 1, b = c = d = 0

$$\Rightarrow h(x)=f(x)$$

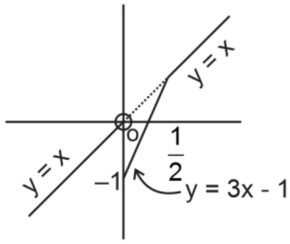
$$h'(0)=f'(0)=\lim_{h\to 0}\frac{\cancel{h}\,|h|\,\text{sum}\frac{1}{h}-0}{\cancel{h}}=0$$

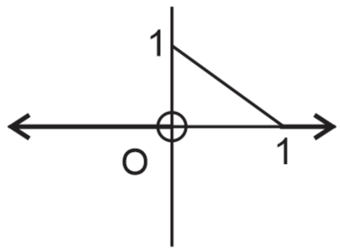
$\therefore$ h is diff. on R.

(R) c = 1, a = b = d = 0

$$\Rightarrow h(x)=x-g(x)=\begin{cases}3x-1, & 0\le x\le\frac{1}{2}\\ x & ,\text{otherwise}\end{cases}$$

From graph, h is many one – onto.





(S)

h(x) = g(x) from graph,

h is into, non-diff., and has range [0,1].