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JEE (ADVANCED) 2018 PAPER-2

[PAPER WITH SOLUTION]

HELD ON SUNDAY 20TH MAY, 2018

MATHEMATICS

SECTION 1 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: **+4**	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: **+3**	If all the four options are correct but ONLY three options are chosen.
Partial Marks	: **+2**	If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	: **+1**	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	: **0**	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	: **–2**	In all other cases.

- **For example :** If If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

1. For any positive integer n, define $f_n:(0,\infty)\to R$ as

$f_n(x)=\sum_{j=1}^{n}\tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$ for all $x\in(0,\infty)$.

(Here, the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$)

Then, which of the following statement(s) is (are) TRUE?

(A) $\sum_{j=1}^{5}\tan^2(f_j(0))=55$

(B) $\sum_{j=1}^{10}\left(1+f_j'(0)\right)\sec^2(f_j(0))=10$

(C) For any fixed positive integer n, $\lim_{x\to\infty}\tan(f_n(x))=\frac{1}{n}$

(D) For any fixed positive integer n, $\lim_{x\to\infty}\sec^2(f_n(x))=1$

1. (D) as x = 0 is not given in the domain

$$\because \quad f_n(x)=\sum_{j=1}^{n}\tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$$

$$f_n(x)=\tan^{-1}(x+n)-\tan^{-1}x$$

$$f_n'(x)=\frac{1}{1+(x+n)^2}-\frac{1}{1+x^2}$$

$$f_n(0)=\tan^{-1}n\Rightarrow\tan^2(\tan^{-1}n)=n^2$$

(A) $$\sum_{j=1}^{5}\tan^2(f_j(0))=\sum_{j=1}^{5}j^2=\frac{5.6.11}{6}=55$$

(B) $$f_n'(0)=\frac{1}{1+n^2}-1\Rightarrow 1+f'_n(0)=\frac{1}{1+n^2}$$

$$\sec^2(f_n(0))=\sec^2(\tan^{-1}n)=1+n^2$$

$$\therefore\ (1+f_n'(0))\sec^2(f_n(0))=\left(\frac{1}{1+n^2}\right)(1+n^2)=1$$

So, $$\sum_{j=1}^{10}(1+f_j'(0))\sec^2(f_j(0))=10$$

(C) $$\lim_{x\to\infty}\tan(f_n(x))=\lim_{x\to\infty}\left(\frac{n}{1+x(n+x)}\right)=0$$

(D) $$\lim_{x\to\infty}\sec^2(f_n(x))=1$$

2. Let T be the line passing through the points P(-2,7) and Q(2,-5). Let F_1 be the set of all pairs of circles (S_1,S_2), such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1,S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R(1,1) be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE?

(A) The point (-2,7) lies in E_1

(B) The point $\left(\frac{4}{5},\frac{7}{5}\right)$ does NOT lie in E_2

(C) The point $\left(\frac{1}{2},1\right)$ lies in E_2

(D) The point $\left(0,\frac{3}{2}\right)$ does NOT lie in E_1

2. **(B,D)**

Let the common tangent to the circles at M intersect the line T at R, then PR = RM = QR

$\therefore$ locus of M is the circle having PQ as diameter excluding points P and Q.

$\therefore$ locus of M is (x + 2) (x -2) + (y-7)(y+5) = 0

i.e. $x^2 + y^2 - 2y - 39 = 0$ and $x \neq \pm 2$

Equation of chord of this circle with mid point (h,k)

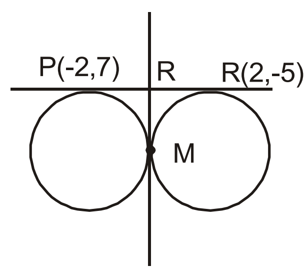
is $hx + ky - (y + k) - 39 = h^2 + k^2 - 2k - 39$.

This chord passes through R(1,1).

$\therefore$ $h + k - 1 - k = h^2 + k^2 - 2k$

$\therefore$ locus of mid point is

$x^2 + y^2 - x - y + 1 = 0$



3. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in R$ and the system of equations (in real variables)

$$\begin{aligned} -x + 2y + 5z &= b_1 \\ 2x - 4y + 3z &= b_2 \\ x - 2y + 2z &= b_3 \end{aligned}$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(A) $x+2y+3z=b_1, 4y+5z=b_2$ and $x+2y+6z=b_3$

(B) $x+y+3z=b_1, 5x+2y+6z=b_2$ and $-2x-y-3z=b_3$

(C) $-x+2y-5z=b_1, 2x-4y+10z=b_2$ and $x-2y+5z=b_3$

(D) $x+2y+5z=b_1, 2x+3z=b_2$ and $x+4y-5z=b_3$

3. **(A,D).**

$\Delta = 0$ So for atteast one solution $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$\Rightarrow b_1 + 7b_2 = 13b_3$ ..... (i)

Option (A) $\Delta \neq 0 \Rightarrow$ Unique solution Option (A) is Correct.

Option (D) $\Delta \neq 0 \Rightarrow$ Unique solution Option (D) is Correct

Option (C) $\Delta = 0 \Rightarrow$ equations are $x-2y+5z=-b_1$

$$x-2y+5z=b_2/2$$

$$x-2y+5z=b_3$$

Three planes are parallel but for all b_1, b_2, b_3 planes are not coincident $\Rightarrow$ Option (C) is wrong.

Option (B) $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 5 & 2 & 2 \\ 2 & 1 & 1 \end{vmatrix} = 0$ also $\Delta_1 = 0$,

for infinite solution Δ_2 & Δ_3 must be $= 0$.

$\Rightarrow \begin{vmatrix} 1 & b_1 & 1 \\ 5 & b_2 & 2 \\ 2 & b_3 & 1 \end{vmatrix} = 0 \Rightarrow -b_1 - b_2 + 3b_3 = 0$ which does not satisfy (1) for all b1, b2, b3

So Option (B) is wrong

4. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin O (0,0) and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE?

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(C) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

4. **(A,C)**

Let the common tangent be y = mx + $\frac{1}{m}$, then $\frac{|\frac{1}{m}|}{\sqrt{m^2+1}} = \frac{1}{\sqrt{2}}$

$\Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow m = \pm 1$

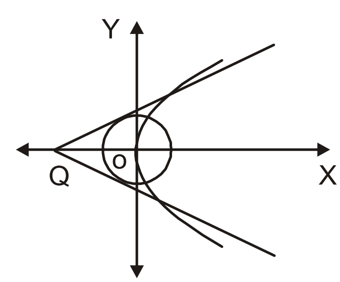
$\therefore$ tangent are y = x + 1 and y = – x – 1.

$\therefore Q \equiv (-1,0)$

$\therefore$ Equation of ellipse is $\frac{x^2}{1} + \frac{y^2}{1/2} = 1.$

$\therefore e = \sqrt{1 - \frac{1/2}{1}} = \frac{1}{\sqrt{2}}$ and length of latus rectum $\frac{2 \times \frac{1}{2}}{1} = 1$

Area of the req. region $= 2\int_{1/\sqrt{2}}^{1} \frac{1}{\sqrt{2}}\sqrt{1-x^2}\,dx$



$= \sqrt{2}\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{\sqrt{2}}}^{1}$

$= \sqrt{2}\left[\frac{\pi}{4} - \left(\frac{1}{4} + \frac{\pi}{8}\right)\right] = \frac{\pi - 2}{4\sqrt{2}}$

5. Let s,t,r be non-zero complex numbers and L be the set of solutions $z = x + iy\ (x, y \in R, i = \sqrt{-1})$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE?

(A) If L has exactly one element, then $|s| \neq |t|$

(B) If |s| = |t|, then L has infinitely many elements

(C) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(D) If L has more than one element, then L has infinitely many elements

5. **(A,C,D)**

$sz + t\bar{z} + r = 0$ ...(1)

$\bar{s}\bar{z} + \bar{t}\,z + \bar{r} = 0$ ...(2)

Eliminating $\bar{z}$ from (1) & (2),

$(s\bar{s} - t\bar{t})z + r\bar{s} - \bar{r}t = 0$

$\Rightarrow (|s|^2 - |t|^2)z = \bar{r}t - r\bar{s}.$

$\therefore$ Unique solution iff $|s| \neq |t|$

Infinite solution iff $|s| = |t|$ and $r\bar{s} = \bar{r}t$

No solution iff $|s| = |t|$ and $r\bar{s} \neq \bar{r}t$

6. Let $f : (0, \pi) \to R$ be a twice differentiable function such that

$$\lim_{t \to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x \text{ for all } x \in (0, \pi).$$

If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE?

(A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$

(B) $f(x) < \frac{x^4}{6} - x^2$ for all $x \in (0, \pi)$

(C) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$

(D) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

6. **(B,C,D)**

$$\lim_{t \to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x$$

by using L'Hospital

$$\lim_{t \to x} \frac{f(x)\cos t - f'(t)\sin x}{1} = \sin^2 x$$

$$\Rightarrow f(x)\cos x - f'(x)\sin x = \sin^2 x$$

$$\Rightarrow -\left(\frac{f'(x)\sin x - f(x)\cos x}{\sin^2 x}\right) = 1$$

$$\Rightarrow -d\left(\frac{f(x)}{\sin x}\right) = 1 \Rightarrow \frac{f(x)}{\sin x} = -x + c$$

Put x = $\frac{\pi}{6}$ & $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$

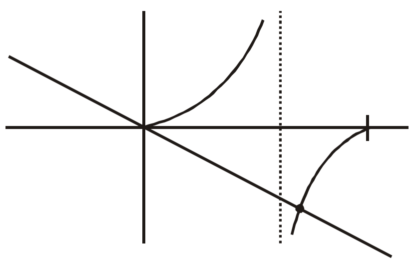
$\therefore c = 0 \Rightarrow f(x) = -x\sin x$

(A) $f\left(\frac{\pi}{4}\right) = \frac{-\pi}{4}\frac{1}{\sqrt{2}}$

(B) f(x) = – xsinx

as sinx > $x - \frac{x^3}{6}, -x\sin x < -x^2 + \frac{x^4}{6}$ $\therefore f(x) < -x^2 + \frac{x^4}{6}\ \forall x \in (0,\pi)$

(C) $f'(x) = -\sin x - x\cos x$



$f'(x) = 0 \Rightarrow \tan x = -x \Rightarrow$ there exist $\alpha \in (0,\pi)$ for which $f'(\alpha) = 0$

(D) $f''(x) = -2\cos x + x\sin x$

$f''\left(\frac{\pi}{2}\right) = \frac{\pi}{2}, f\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$

$f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

SECTION 2 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place;** e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+3** If ONLY the correct numerical value is entered as answer.

Zero Marks **: 0** In all other cases.

7. The value of the integral $\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}}dx$ is __________.

7. **(2)**

$$I=\int_0^{1/2}\frac{\left(1+\sqrt{3}\right)dx}{\left\{(x+1)^2(1-x)^6\right\}^{1/4}}=\int_0^{1/2}\frac{\left(1+\sqrt{3}\right)dx}{\left\{(x+1)^2\left(\frac{1-x}{1+x}\right)^{3/2}\right\}} \qquad \left(\text{Put } \frac{1-x}{1+x}=t,\ \frac{-2}{(1+x)^2}dx=dt\right)$$

$$I=\frac{1}{2}\int_{1/3}^{1}\frac{\left(1+\sqrt{3}\right)dt}{t^{3/2}}$$

$$I=\left(\frac{1+\sqrt{3}}{2}\right)\left(\frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\right)_{1/3}^{1}$$

$$=-\left(\frac{1+\sqrt{3}}{2}\right)\left(1-\sqrt{3}\right).2=2$$

8. Let P be a matrix of order 3 × 3 such that all the entries in P are from the set {–1, 0, 1}. Then, the maximum possible value of the determinant of P is _____ .

8. **(4)**

$$\begin{vmatrix} A & B & C \\ D & E & F \\ G & H & I \end{vmatrix}=$$

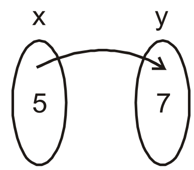
$$A(EI-HF)-B(DI-GF)+C(DH-EG)$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}=4$$

It's maximum cannot to equal to 5. If any of the entry is zero it effects at two terms, which results the sum as 4 again.

9. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta-\alpha)$ is _____ .

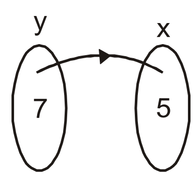
9. (119)



Number of one-one function $\Rightarrow {}^7C_5 \cdot \underline{|5} \Rightarrow 2520$

Hence, $\alpha = 2520$

Now β is the onto function



$\Rightarrow 5^7 - {}^5C_1(5-1)^7 + {}^5C_2(5-2)^7 - {}^5C_3(5-3)^7 + {}^5C_4(5-4)^7 - {}^5C_5(5-5)^7$

$\Rightarrow 5^7 - 5(4)^7 + 10(3)^7 - 10(2)^7 + 5$

$\Rightarrow 78125 - 81920 + 21870 - 1280 + 5$

$\Rightarrow 100000 - 83200 = 16800$

Now, $\frac{1}{\underline{|5}}(\beta - \alpha) = \frac{1}{120}(16800 - 2520) = 119$

10. Let f: R → R be a differentiable function with f (0) = 0. If y = f (x) satisfies the differential equation $\frac{dy}{dx} = (2+5y)(5y-2)$, then the value of $\lim_{x\to-\infty} f(x)$ is _____.

10. (0.4)

$$\frac{dy}{dx} = (5y+2)(5y-2)$$

$$\frac{1}{25}\int \frac{dy}{(y+2/5)(y-2/5)} = \int dx$$

$$\Rightarrow \frac{1}{25}.\frac{5}{4}\ln\left|\frac{y-2/5}{y+2/5}\right| = x + c$$

$$\Rightarrow \frac{1}{20}\ln\left|\frac{5y-2}{5y+2}\right| = x + c$$

at $x = 0, y = 0 \Rightarrow c = 0$

$\therefore \ \dfrac{2-5y}{2+5y} = e^{20x}$

$\dfrac{2-5y}{2+5y} = e^{20x} \therefore \lim\limits_{x\to\infty} e^{20x} = 0$

$\lim\limits_{x\to\infty} y = 2/5 = 0.4$

11. Let f: R → R be a differentiable function with f (0) = 1 and satisfying the equation

$f(x+y) = f(x)f'(y) + f'(x)f(y)$ for all x, y $\in$R.

Then, the value of $\log_e$ (f(4)) is _____.

11. (2)

$\because f(x+y) = f(x)f'(y) + f'(x).f(y) \quad ...(1)$

Put $x = y = 0$

$f(0) = 2f'(0) \Rightarrow f'(0) = 1/2$

Now, Put $y = 0$

$f(x) = f(x).f'(0) + f'(x)f(0)$

$f(x) = \lambda e^{x/2} \Rightarrow f(x) = e^{x/2}$ (as $f(0) = 1$)

Now,

$\ln(f(x)) = x/2 \Rightarrow \ln f(4) = 2$

12. Let P be a point in the first octant, whose image Q in the plane x + y = 3 (that is, the line segment PQ is perpendicular to the plane x + y = 3 and the mid-point of PQ lies in the plane x + y = 3) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is _____ .

12. (8)

$P(\alpha, \beta, \gamma)$

$R(\alpha, \beta, -\gamma)$

$Q \equiv \dfrac{x-\alpha}{1} = \dfrac{y-\beta}{1} - \dfrac{z-\gamma}{0} = \dfrac{-2(\alpha+\beta-\gamma)}{2}$

$x = 3-\beta,\ y = 3-\alpha,\ x = \gamma$

$Q(3-\beta, 3-\alpha, \gamma)$ lies on $z-$axis

$\beta = 3,\ \alpha = 3$

$P(3,3,\gamma)$ distance from x – axis is 5

$9+\gamma^2=25$

$\gamma^2=16$

$\gamma=4$

P (3,3,4)

R (3,3,–4)

PR = 8

13. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O(0,0,0) is the origin. Let $S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p}=\overrightarrow{SP},\vec{q}=\overrightarrow{SQ},\vec{r}=\overrightarrow{SR}$ and $\vec{t}=\overrightarrow{ST}$, then the value of $\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|$ is ______ .

13. (1/2)

$$\vec{p}=\frac{1}{2}\left(\hat{i}-\hat{j}-\hat{k}\right),\vec{q}=\frac{1}{2}\left(-\hat{i}+\hat{j}-\hat{k}\right),\vec{r}=\frac{1}{2}\left(-\hat{i}-\hat{j}+\hat{k}\right),\vec{t}=\frac{1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|=\frac{1}{16}\left|\left(2\hat{i}+2\hat{j}\right)\right|\times\left|\left(-2\hat{i}+2\hat{j}\right)\right|=\frac{1}{2}\left|\hat{k}\right|=\frac{1}{2}$$

14. Let $X=({}^{10}C_1)^2+2({}^{10}C_2)^2+3({}^{10}C_3)^2+\ldots+10({}^{10}C_{10})^2$, where ${}^{10}C_r, r\in\{1,2,\ldots,10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is .

14. 646

$$X=\sum_{r=1}^{10} r\,{}^{10}C_r^2=\sum_{r=1}^{10} r.\frac{10}{r}.\,{}^{9}C_{r-1}.\,{}^{10}C_r$$

$$=10\sum_{r=1}^{10} {}^{9}C_{r-1}.\,{}^{10}C_{10-r}$$

$$=10.\,{}^{19}C_9 \qquad \therefore \frac{X}{1430}=646$$

SECTION 3 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **TWO (02)** matching lists: **LIST-I** and **LIST-II.**
- **FOUR** options are given representing matching of elements from **LIST-I** and **LIST-II**. **ONLY ONE** of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme:

  Full Marks : **+3** If ONLY the option corresponding to the correct matching is chosen.

  Zero Marks **: 0** If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : **–1** In all other cases.

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15. Let $E_1 = \left\{x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$

$E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{ is a real number}\right\}.$

(Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.)

Let $f : E_1 \rightarrow R$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$

and $g : E_2 \rightarrow R$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$.

List –I	List-II
P. The range of f is	1. $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
Q. The range of g contains	2. (0, 1)
R. The domain of f contains	3. $\left[-\frac{1}{2}, \frac{1}{2}\right]$
S. The domain of g is	4. $(-\infty, 0) \cup (0, \infty)$
	5. $\left(-\infty, \frac{e}{e-1}\right]$
	6. $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is :

(A) $P \to 4; Q \to 2; R \to 1; S \to 1$

(B) $P \to 3; Q \to 3; R \to 6; S \to 5$

(C) $P \to 4; Q \to 2; R \to 1; S \to 6$

(D) $P \to 4; Q \to 3; R \to 6; S \to 5$

15. (A)

$f(x) = \log \frac{x}{x-1}$

Domain $\frac{x}{x-1} > 0 \Rightarrow x \in (-\infty, 0) \cup (1, \infty)$

Range : range of $\frac{x}{x-1}$ is R-{1}

$\therefore$ range of f is R-{0}

$g(x) = \sin^{-1}\left(\log \frac{x}{x-1}\right)$

Domain : $-1 \le \log \frac{x}{x-1} \le 1$

$\Rightarrow \frac{1}{e} \le \frac{x}{x-1} \le e$

$\Rightarrow \frac{1}{e} - 1 \le \frac{1}{x-1} \le e - 1$

$\Rightarrow x - 1 \le \frac{e}{1-e}$ or $x - 1 \ge \frac{1}{e-1}$

$\Rightarrow x \in \left(-\infty, -\frac{1}{e-1}\right] \cup \left[\frac{e}{e-1}, \infty\right)$

Range : range of $\log \frac{x}{x-1}$ is R-{0}

$\therefore$ range of g is [-1,1]-{0}.

16. In a high school, a committee has to be formed from a group of 6 boys M_1, M_2,M_3,M_4,M_5,M_6 and 5 girls G_1,G_2,G_3,G_4,G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M_1 and G_1 are NOT in the committee together.

LIST–I	**LIST–II**
P. The value of α_1 is	1. 136
Q. The value of α_2 is	2. 189
R. The value of α_3 is	3. 192
S. The value of α_4 is	4. 200
	5. 381
	6. 461

The correct option is:

(A) P $\rightarrow$ 4; Q $\rightarrow$ 6; R $\rightarrow$ 2; S $\rightarrow$ 1

(B) P $\rightarrow$ 1; Q $\rightarrow$ 4; R $\rightarrow$ 2; S $\rightarrow$ 3

(C) P $\rightarrow$ 4; Q $\rightarrow$ 6; R $\rightarrow$ 5; S $\rightarrow$ 2

(D) P $\rightarrow$ 4; Q $\rightarrow$ 2; R $\rightarrow$ 3; S $\rightarrow$ 1

16. (C)

$\alpha_1 = {}^6C_3 \times {}^5C_2 = 200$ (3 boys + 2 girls)

$\alpha_2 = \sum_{r=1}^{5} {}^6C_r \cdot {}^5C_r$ (equal number of boys an girls)

$= {}^{11}C_5 - {}^6C_0 \cdot {}^5C_0 = 461$

$\alpha_3 = {}^{11}C_5 - ({}^6C_5 \cdot {}^5C_0 + {}^6C_4 \cdot {}^5C_1) = 381$

α_4 = Total number of ways with at least 2 girls- number of ways in which M_1 and G_1 are together

$= {}^6C_2 \times {}^5C_2 + {}^6C_1 \times {}^5C_3 + {}^6C_0 \times {}^5C_4 - ({}^5C_0 \times {}^4C_2 + {}^5C_1 \times {}^4C_1)$

= 150+ 60 +5 – (6 + 20) = 189

17. Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where a > b > 0, be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of $60°$ at one of its vertices N. Let the area of the triangle LMN be $4\sqrt{3}$.

LIST–I	LIST-II
P. The length of the conjugate axis of H is	1. 8
Q. The eccentricity of H is	2. $\frac{4}{\sqrt{3}}$
R. The distance between the foci of H is	3. $\frac{2}{\sqrt{3}}$
S. The length of the latus rectum of H is	4. 4

The correct option is:

(A) P → 4; Q → 2; R → 1; S → 3

(B) P → 4; Q → 3; R → 1; S → 2

(C) P → 4; Q → 1; R → 3; S → 2

(D) P → 3; Q → 4; R → 2; S → 1

17. (B)

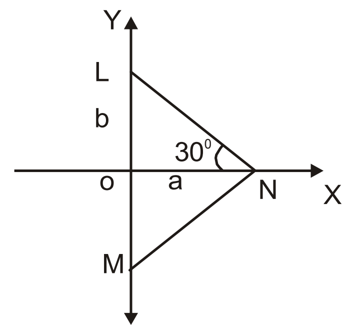
Here $\tan 30° = \frac{b}{a} \Rightarrow \frac{b}{a} = \frac{1}{\sqrt{3}}$ ....(1)

And area of ΔLMN

$= \frac{1}{2} \times 2b \times a = 4\sqrt{3}$

$\Rightarrow ab = 4\sqrt{3}$ ....(2)

From (1) & (2), $a^2 = 12$ and $b^2 = 4$



(P) Length of conjugate axis = 2b = 4

(Q) eccentricity $= \sqrt{1+\frac{b^2}{a^2}} = \frac{2}{\sqrt{3}}$

(R) The distance between foci $= 2ae = 2\sqrt{12} \times \frac{2}{\sqrt{3}} = 8$

(S) The length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 4}{\sqrt{12}} = \frac{4}{\sqrt{3}}$

18. Let $f_1 : R \to R$, $f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$, $f_3 : \left(-1, e^{\frac{\pi}{2}} - 2\right) \to R$ and $f_4 : R \to R$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1-e^{-x^2}}\right)$

(ii) $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iii) $f_3(x) = \left[\sin\left(\log_e(x+2)\right)\right]$, where, for $t \in R$, $[t]$ denotes the greatest integer less than or equal to t,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

LIST- I		LIST-II	
P.	The function f_1 is	1.	**NOT** continuous at x = 0
Q.	The function f_2 is	2.	Continuous at x = 0 and **NOT** differentiable at x = 0
R.	The function f_3 is	3.	Differentiable at x = 0 and its derivative is **NOT** continuous at x = 0
S.	The function f_4 is	**4.**	differentiable at x = 0 and its derivative is continuous at x = 0

The correct option is:

(A) P → 2; Q → 3; R → 1; S → 4

(B) P → 4; Q → 1; R → 2; S → 3

(C) P → 4; Q → 2; R → 1; S → 3

(D) P → 2; Q → 1; R → 4; S → 3

18. (D)

(P) $\lim_{x\to 0} f_1(x) = 0 = f_1(0)$

$$f_1'(0) = \lim_{h\to 0} \frac{\sin\sqrt{1-e^{-h^2}} - 0}{h}$$

$$= \lim_{h\to 0} \frac{\sin\sqrt{1-e^{-h^2}}}{\sqrt{1-e^{-h^2}}} \cdot \sqrt{\frac{e^{-h^2}-1}{-h^2}} \cdot \frac{|h|}{h}$$

$$= 1.\lim_{h\to 0} \frac{|h|}{h}$$

Which does not exist.

$\therefore$ f_1 is continuous but not differentiable at x = 0.

(Q) $\lim_{x\to 0} f_2(x) = \lim_{x\to 0} \cdot \left|\frac{\sin x}{x}\right| \cdot \frac{x}{\tan^{-1} x} \cdot \frac{|x|}{x}$

$$= \lim_{x\to 0} \frac{|x|}{x}$$

which does not exist.

$\therefore$ f_2 is not continues at x = 0.

(R) $f_3(x) = [\sin(\log(x+2))]$

$$= 0 \ \forall x \in \left[-1, \frac{1}{2}\right]$$

$\therefore$ f_3 has continuous derivative at x = 0.

(S) $f_4'(0) = \lim_{h\to 0} \dfrac{h^2 \sin\frac{1}{h} - 0}{h} = \lim_{h\to 0} h \sin\frac{1}{h} = 0$

Also $f_4'.(x) = \begin{cases} 2x\sin\frac{1}{x} + x^2\cos\frac{1}{x}.\frac{-1}{x^2}, & x \neq 0 \\ 0 & x = 0 \end{cases}$,

f_4 is differentiable at x = 0 but its derivative is not continuous at x = 0.