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# JEE (ADVANCED) 2018 PAPER-2

## [PAPER WITH SOLUTION]

**HELD ON SUNDAY 20TH MAY, 2018**

# PHYSICS

## SECTION 1 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.

Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.

Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.

Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : **–2** In all other cases.

- **For example :** If If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

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**1.** A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where $\gamma$ is a positive constant of appropriate dimensions. Which of the following statements is (are) true?

(A) The force applied on the particle is constant

(B) The speed of the particle is proportional to time

(C) The distance of the particle from the origin increases linearly with time

(D) The force is conservative

**1.** **(A,B,D)**

**Sol.** Kinetic energy, $K = \frac{1}{2}mv^2$

By differentiating the above equation w.r.t. time

$$\frac{dk}{dt} = \frac{1}{2}m(2v)\frac{dv}{dt} = mv\frac{dv}{dt}$$

$$\gamma t = \frac{mvdv}{dt} \qquad \left[\frac{dk}{dt} = \gamma t \ \text{ given in the question}\right]$$

$$\int_0^v vdv = \int_0^t \frac{\gamma t}{m}dt$$

$$\frac{v^2}{2} = \frac{\gamma}{m}\frac{t^2}{2}$$

$v = \sqrt{\frac{\gamma}{m}}t$, The speed of the particle is proportional to time, (B)

$$\frac{dv}{dt} = \text{constant}$$

$F = m\frac{dv}{dt} = \text{constant}$, (A)

We know that all constant force is conservative, (D)

**2.** Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity $u_0$. Which of the following statements is (are) true?

(A) The resistive force of liquid on the plate is inversely proportional to h

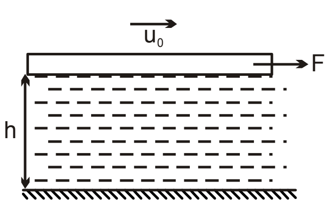
(B) The resistive force of liquid on the plate is independent of the area of the plate

(C) The tangential (shear) stress on the floor of the tank increases with $u_0$

(D) The tangential (shear) stress on the plate varies linearly with the viscosity $\eta$ of the liquid

**2.** **(A,C,D)**

**Sol.**




Viscous force on the plate

$$F = -\eta A\frac{dv}{dh}$$

(–)ve sign shows the direction of viscous force.

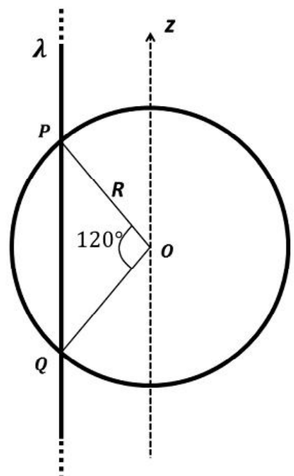
$|F| = \eta A \frac{u_0}{h}$

The resistive force of liquid on the plate is inversely proportional to h, (A)

The tangential stress $= \frac{F}{A} = \eta \frac{u_0}{A} \propto u_0$, (C)

The tangential stress $= \frac{F}{A} \propto \eta$, (D)

**3.** An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density $\lambda$. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is $\epsilon_0$. Which of the following statements is (are) true?




(A) The electric flux through the shell is $\sqrt{3}\, R\lambda / \epsilon_0$

(B) The z-component of the electric field is zero at all the points on the surface of the shell

(C) The electric flux through the shell is $\sqrt{2}\, R\lambda / \epsilon_0$

(D) The electric field is normal to the surface of the shell at all points

**3.** **(A, B)**

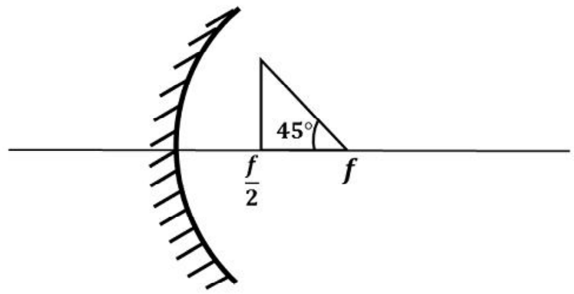
**Sol.** The charge inclosing the spherical shell $= \lambda(\sqrt{3}R)$

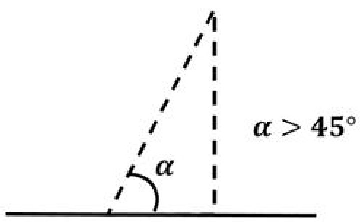
The electric flux through the shell $= \frac{Q_{inside}}{\epsilon_0}$

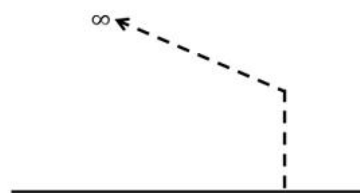
$\phi = \frac{\lambda\sqrt{3}R}{\epsilon_0}$ [Option (A) is correct]

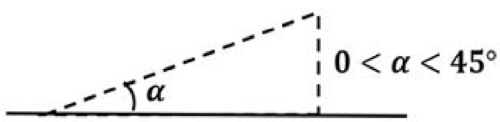
The z-component of the electric field = 0 [Option (B) is correct]

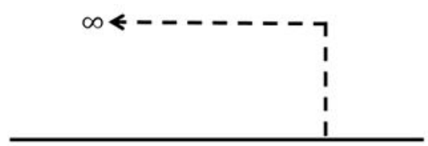
**4.** A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.)




(A) 


(B) 


(C) 


(D) 


**Ans.**

**Sol.** Let us consider a point $P(-x_0, y_0)$

$\therefore y_0 = (f - x_0)$

Magnification

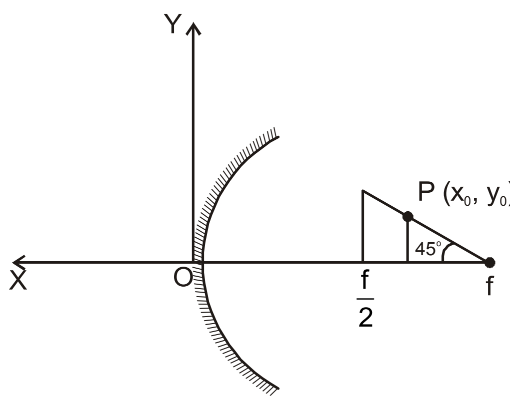
$$m_y = \frac{-f}{-f-(-x_0)} = \frac{f}{f-x_0}$$

Let co-ordinate of image be $(x_i, y_i)$

$$\therefore\ y_i = m_y\, y_0 = \frac{f}{f-x_0} \times y_0 = f$$

$\therefore\ y_i = f$

Thus locus of image will be parallel to x - axis.



**5.** In a radioactive decay chain, ${}^{232}_{90}Th$ nucleus decays to ${}^{212}_{82}Pb$ nucleus. Let $N_\alpha$ and $N_\beta$ be the number of $\alpha$ and $\beta^-$ particles, respectively, emitted in this decay process. Which of the following statements is (are) true?

(A) $N_\alpha = 5$ (B) $N_\alpha = 6$ (C) $N_\beta = 2$ (D) $N_\beta = 4$

**Ans.** **(A,C)**

**Sol.** ${}^{232}_{90}Th \longrightarrow {}^{212}_{82}Pb + n_1\ {}^{4}_{2}\alpha + n_2\ {}^{0}_{-1}B$

Now, $90 = 82 + 2n_1 - n_2$ ..(i)

and $232 = 212 + 4n_1$ ...(ii)

$\Rightarrow$ $n_1 = 5$

also $n_2 = 2$ from (i)

**6.** In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true?

(A) The speed of sound determined from this experiment is 332 m $s^{-1}$

(B) The end correction in this experiment is 0.9 cm

(C) The wavelength of the sound wave is 66.4 cm

(D) The resonance at 50.7 cm corresponds to the fundamental harmonic

**Ans.** **(A,C)**

**Sol.** We have

$n\frac{\lambda}{2} + \frac{\lambda}{4} = 50.7 + e$ ....(i) where e is end correction

also $(n+1)\frac{\lambda}{2} + \frac{\lambda}{4} = 83.9 + e$ ...(ii)

$\Rightarrow \frac{\lambda}{2} = 33.2$ $\Rightarrow \lambda = 66.4$ cm

Now $V = \frac{500 \times 66.4}{100}$ $\Rightarrow V = 332$ m/s

$\Rightarrow$ (A) & (C) are correct

Now putting $\lambda = 66.4$ cm in equation (i)

we have

$n(33.2) + 16.6 = 50.7 + e$

$\Rightarrow e = n(33.2) - 34.1$

Putting n = 1 $\Rightarrow e = -0.9$ (Not possible)

Putting n = 2 $\Rightarrow e = 32.3$ (Not possible)

## SECTION 2 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place;** e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+3** If ONLY the correct numerical value is entered as answer.

  Zero Marks **: 0** In all other cases.

**7.** A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4 kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where $v_0$ is a constant and $\tau$ = 4 s. The displacement of the block, in metres, at t = $\tau$ is __________. Take $e^{-1}$ = 0.37.

**Ans. 6.30 m**

**Sol.** Impulse = $\Delta$ momentum

$\Rightarrow$ 1 = 0.4 V

$\Rightarrow \quad v = \frac{5}{2} m/s \qquad \Rightarrow \quad v_0 = \frac{5}{2} m/s$

Now $v = v_0 e^{-t/4}$

$\Rightarrow \int dx = \frac{5}{2}\int_0^4 e^{-\frac{t}{4}} dt$

$\Rightarrow x = -\frac{5}{2} \times 4 \left(e^{-1} - 1\right)$

$\Rightarrow \quad x = 10\,(1 - e^{-1}) = 10 \times 0.63 = 6.3\text{ m}$

**8.** A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is ___________.

**Ans. 30 m**

**Sol.** $H \propto v^2 \sin^2\theta$ As $H = \frac{v^2 \sin^2\theta}{2g}$

$\Rightarrow \quad 120 \propto v^2 \frac{1}{2}$

and $H \propto \frac{v^2}{2} \times \frac{1}{4}$

$\because$ Kinetic energy becomes half

$\Rightarrow \quad \frac{120}{H} = 4 \qquad \Rightarrow \quad H = 30\text{ m}$

**9.** A particle, of mass $10^{-3}$ kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin\omega t\,\hat{i}$, where $E_0$ = 1.0 N $C^{-1}$ and $\omega$ = $10^3$ rad $s^{-1}$. Consider the effect of only the electrical force on the particle. Then the maximum speed, in m $s^{-1}$, attained by the particle at subsequent times is ____________.

**Ans. 2 m/s**

**Sol.** We have F = qE

$\Rightarrow \quad m\frac{dV}{dt} = qE_0 \sin\omega t$

$\Rightarrow \quad 10^{-3}\int dV = \int_0^{T/2} \sin\omega t\, dt$

$\Rightarrow \quad 10^{-3}V = \left[\frac{-\cos\omega t}{\omega}\right]_0^{T/2}$

$\Rightarrow \quad (10^{-3})V = \frac{1}{\omega}(1-\cos\pi)$

$\Rightarrow \quad v = \frac{2}{10^{-3}\omega}$ Putting $\omega = 10^3$ rad/s

$\Rightarrow \quad v = 2$ m/s

**10.** A moving coil galvanometer has 50 turns and each turn has an area 2 × $10^{-4}$ $m^2$. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is $10^{-4}$ N m $rad^{-1}$. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 –1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is__________.

**Ans. (5.55)**

**Sol.** At the equilibrium condition

$Ni_gAB = C\theta$

$i_g = \frac{C\theta}{NAB} = \frac{10^{-4}\times 0.2}{50\times 2\times 10^{-4}\times 0.02} = 0.1A$

$i_gG = (i - i_g)S$ [since they are connected in parallel]

$0.1\times 50 = (1-0.1)S$

$S = \frac{50}{9}\Omega = 5.55$

**11.** A steel wire of diameter 0.5 mm and Young's modulus $2 \times 10^{11}$ N $m^{-2}$ carries a load of mass M. The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2 kg, the vernier scale division which coincides with a main scale division is __________. Take g= 10 $ms^{-2}$ and $\pi = 3.2$.

**Ans.** **(3)**

**Sol.** $$\Delta\ell = \frac{F\ell}{AY} = \frac{1.2 \times 10 \times 1}{\pi \times \left(\frac{5 \times 10^{-4}}{2}\right)^2 \times 2 \times 10^{11}}$$

Solving this we get

$$\Delta\ell = 0.3 \text{ mm}$$

Since least count of vernier caliper is 0.1mm, So, third marking of vernier scale will coincide with main scale.

**12.** One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 J $mol^{-1}$ $K^{-1}$, the decrease in its internal energy, in Joule, is__________.

**Ans.** **(900)**

**Sol.** For adiabatic expansion

$$TV^{\gamma-1} = T_f (8V)^{\gamma-1}$$

$$T_f = \frac{T}{(8)^{\frac{5}{3}-1}} = \frac{T}{4}$$

$$\Delta U = nC_v\Delta T = 1 \times \frac{3}{2}R \times \left(\frac{T}{4} - T\right)$$

$$= 1 \times \frac{3}{2}R \times \left(-\frac{3T}{4}\right)$$

$$= 1 \times \frac{3}{2}8 \times \left(-\frac{3}{4} \times 100\right)$$

$$\Delta U = -900 \text{ J}$$

Decrease in internal energy is 900J

**13.** In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is __________. Mass of the electron $m_e = 9 \times 10^{-31}$ kg and 1.0 eV = $1.6 \times 10^{-19}$ J.

**Ans.** **(24.00)**

**Sol.** Number of electrons emitted $N = \frac{200\,W}{6.25\,eV}$

Rate of change of linear momentum of electrons $= F = N\sqrt{2mK}$

$$F = \frac{200}{1.6 \times 6.25 \times 10^{-19}} \times \sqrt{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19} \times 500}$$

F = 24.00

**14.** Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is __________.

**Ans.** **(3)**

**Sol.** $$13.6\mathbb{Z}^2\left(1-\frac{1}{4}\right) = 74.8 + 13.6\mathbb{Z}^2\left(\frac{1}{4}-\frac{1}{9}\right)$$

$$\Rightarrow 13.6 \times Z^2\left(\frac{11}{18}\right) = 74.8$$

$$\Rightarrow Z^2 = 9$$

$$\Rightarrow Z^2 = 3$$

# SECTION 3 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **TWO (02)** matching lists: **LIST-I** and **LIST-II.**
- **FOUR** options are given representing matching of elements from **LIST-I** and **LIST-II**. **ONLY ONE** of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme:

  Full Marks : **+3** If ONLY the option corresponding to the correct matching is chosen.

  Zero Marks **: 0** If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : **–1** In all other cases.

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**15.** The electric field E is measured at a point P(0, 0, d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

| **List-I** | **List-II** |
|---|---|
| P. E is independent of d | 1. A point charge Q at the origin |
| Q. $E \propto \frac{1}{d}$ | 2. A small dipole with point charges Q at (0, 0, *l*) and – Q at (0, 0, –*l*). Take $2l << d$ |
| R. $E \propto \frac{1}{d^2}$ | 3. An infinite line charge coincident with the x-axis, with uniform linear charge density $\lambda$ |
| S. $E \propto \frac{1}{d^3}$ | 4. Two infinite wires carrying uniform linear charge density parallel to the x- axis. The one along (y = 0, z = *l*) has a charge density +$\lambda$) and the one along (y = 0, z = –*l*) has a charge density – $\lambda$. Take 2*l* j” d |
| | 5. Infinite plane charge coincident with the xy-plane with uniform surface charge density |

(A) P $\rightarrow$ 5; Q $\rightarrow$ 3, 4; R $\rightarrow$ 1; S $\rightarrow$ 2

(B) P $\rightarrow$ 5; Q $\rightarrow$ 3; R $\rightarrow$ 1, 4; S $\rightarrow$ 2

(C) P $\rightarrow$ 5; Q $\rightarrow$ 3; R $\rightarrow$ 1, 2; S $\rightarrow$ 4

(D) P $\rightarrow$ 4; Q $\rightarrow$ 2, 3; R $\rightarrow$ 1; S $\rightarrow$ 5

**Ans.** **(B)**

**Sol.** (1) Electric field at P due to point charge Q

$$=\frac{KQ}{d^2}$$

Ans- $(1)\rightarrow R$

(2) Electric field due to dipole at axial position

$$E=\frac{2KP}{r^3}=\frac{2KQ(2\ell)}{d^3}$$

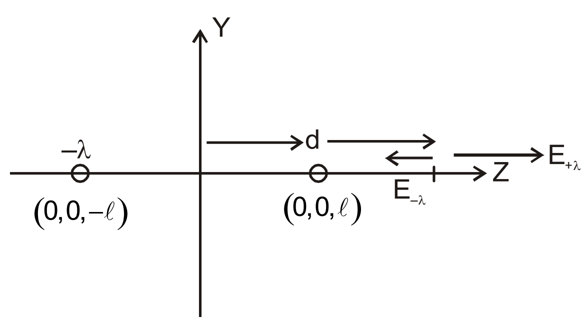
Ans. $(2)\rightarrow S$

(3) Electric field due to uniform line charge at a perpendicular distance of d

$$E=\frac{2K\lambda}{d}$$

Ans. $(3)\rightarrow Q$

(4)




Two infinits line charge parallel to x axis

Net electric field at P due to infinite line cleape

$$=E_{+\lambda}-E_{-\lambda}$$

$E_{+\lambda}$ electric field due line charge density $+\lambda$

$E_{-\lambda}$ electric field due line charge density $-\lambda$

$$=\frac{2K\lambda}{d-\ell}-\frac{2K\lambda}{d+\ell}$$

$$=2K\lambda\left[\frac{(d+\ell)-(d-\ell)}{d^2-\ell^2}\right]$$

$$=\frac{2K\lambda 2\ell}{d^2-\ell^2}$$

$= \dfrac{4K\lambda\ell}{d^2 - \ell^2}$

$d >> \ell$

Net electric field at P

$= \dfrac{4K\lambda\ell}{d^2}$

$E \,\alpha\, \dfrac{1}{d^2}$

$(4) \rightarrow R$

(5) Electric field due to uniformally charge

$\text{Surface} = \dfrac{\sigma}{2\varepsilon_0} = \text{constant}$

Ans. (5)

Therefore Ans. (P)

$P \rightarrow 5;\ \ Q \rightarrow 3;\ R \rightarrow 1,4;\ \ S \rightarrow 2$

**16.** A planet of mass M, has two natural satellites with masses $m_1$ and $m_2$. The radii of their circular orbits are $R_1$ and $R_2$ respectively. Ignore the gravitational force between the satellites. Define $v_1$, $L_1$, $K_1$ and $T_1$ to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and $v_2$, $L_2$, $K_2$ and $T_2$ to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

| **List-I** | **List-II** |
|---|---|
| P. $\dfrac{v_1}{v_2}$ | 1. $\dfrac{1}{8}$ |
| Q. $\dfrac{L_1}{L_2}$ | 2. 1 |
| R. $\dfrac{K_1}{K_2}$ | 3. 2 |
| S. $\dfrac{T_1}{T_2}$ | 4. 8 |

(A) $P \rightarrow 4;\ Q \rightarrow 2;\ R \rightarrow 1;\ S \rightarrow 3$

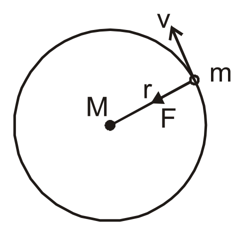
(B) $P \rightarrow 3;\ Q \rightarrow 2;\ R \rightarrow 4;\ S \rightarrow 1$

(C) $P \rightarrow 2;\ Q \rightarrow 3;\ R \rightarrow 1;\ S \rightarrow 4$

(D) $P \rightarrow 2;\ Q \rightarrow 3;\ R \rightarrow 4;\ S \rightarrow 1$

**Ans.** **(B)**

**Sol.** Let a satellite of mass m is rotating about planet of mass M.
gravitational force provide centripital acceleration.




$$F = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}},\ \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = 2 \qquad \Rightarrow (P \rightarrow 3)$$

$\rightarrow$ Kinetic energy of particle $(K) = \frac{1}{2}mv^2$

$$= \frac{GMm}{2r}$$

$$\frac{K_1}{K_2} = \frac{m_1}{m_2}\frac{r_2}{r_1} = (2)\,(4) = 8 \qquad \Rightarrow (R \rightarrow 4)$$

Angular momentum

L = rmv

$$\frac{L_1}{L_2} = \frac{r_1}{r_2}\cdot\frac{m_1}{m_2}\cdot\frac{v_1}{v_2} = \frac{1}{4}.2.2$$

$$\frac{L_1}{L_2} = 1$$

$\Rightarrow (Q \rightarrow 2)$

Time period , $T = \frac{2\pi r}{v}$

$$\frac{T_1}{T_2} = \frac{r_1}{r_2}\frac{v_2}{v_1}$$

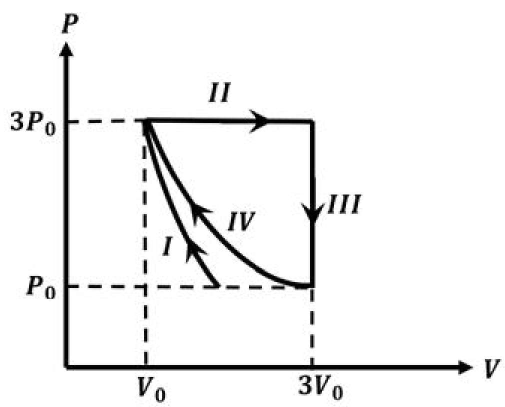
$$= \frac{1}{4}\cdot\frac{1}{2}$$

$$= \frac{1}{8}$$

$$\frac{T_1}{T_2} = \frac{1}{8}$$

$\Rightarrow (S \rightarrow 1)$

**17.** One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-1 with the corresponding statements in List-II.




| List-I | | | List-II |
|---|---|---|---|
| P. | In process I | 1. | Work done by the gas is zero |
| Q. | In process II | 2. | Temperature of the gas remains unchanged |
| R. | In process III | 3. | No heat is exchanged between the gas and its surroundings |
| S. | In process IV | 4. | Work done by the gas is $6P_0V_0$ |

(A) P $\rightarrow$ 4; Q $\rightarrow$ 3; R $\rightarrow$ 1; S $\rightarrow$ 2

(B) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 4

(C) P $\rightarrow$ 3; Q $\rightarrow$ 4; R $\rightarrow$ 1; S $\rightarrow$ 2

(D) P $\rightarrow$ 3; Q $\rightarrow$ 4; R $\rightarrow$ 2; S $\rightarrow$ 1

**Ans. (C)**

**Sol.** P $\rightarrow$ process I $\rightarrow$ adiabatic $\rightarrow$ (3) No heat exchanged

Q $\rightarrow$ process II $\rightarrow$ isobaric $\rightarrow$ (4) Work $= P\Delta V = 6P_0V_0$

R $\rightarrow$ process III $\rightarrow$ isochoric $\rightarrow$ (1) Work = 0

S $\rightarrow$ process IV $\rightarrow$ isothermal $\rightarrow$ (2) Temperature of the gas remains uncharged

**18.** In the List-I below, four different paths of a particle are given as functions of time. In these functions, $\alpha$ and $\beta$ are positive constants of appropriate dimensions and $\alpha \neq \beta$. In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned: $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are **conserved for that path.**

| **List-I** | | **List-II** | |
|---|---|---|---|
| P. | $\vec{r}(t) = \alpha t\,\hat{i} + \beta t\,\hat{j}$ | 1. | $\vec{p}$ |
| Q. | $\vec{r}(t) = \alpha \cos\omega t\,\hat{i} + \beta \sin\omega t\,\hat{j}$ | 2. | $\vec{L}$ |
| R. | $\vec{r}(t) = \alpha\left(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}\right)$ | 3. | K |
| S. | $\vec{r}(t) = \alpha t\,\hat{i} + \frac{\beta}{2}t^2\,\hat{j}$ | 4. | U |
| | | 5. | E |

(A) P $\rightarrow$ 1, 2, 3, 4, 5; Q $\rightarrow$ 2, 5; R $\rightarrow$ 2, 3, 4, 5; S $\rightarrow$ 5

(B) P $\rightarrow$ 1, 2, 3, 4, 5; Q $\rightarrow$ 3, 5; R $\rightarrow$ 2, 3, 4, 5; S $\rightarrow$ 2, 5

(C) P $\rightarrow$ 2, 3, 4; Q $\rightarrow$ 5; R $\rightarrow$ 1, 2, 4; S $\rightarrow$ 2, 5

(D) P $\rightarrow$ 1, 2, 3, 5; Q $\rightarrow$ 2, 5; R $\rightarrow$ 2, 3, 4, 5; S $\rightarrow$ 2, 5

**Ans.** **(A)**

**Sol.** (P) Position vector of particle

$$\vec{r}(t) = \alpha t\hat{i} + \beta t\,\hat{j}$$

velocity of particle at time t

$$\vec{v} = \frac{d\vec{r}(t)}{dt} = \alpha\hat{i} + \beta\hat{j}$$

acceleration of particle at time t

$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{o}$$

Force on a particle is equal to zero

$\rightarrow$ we known that

$$\vec{F} = \frac{d\vec{P}}{dt} = 0$$

$$\vec{P} = \text{constant}$$

$\rightarrow$ Torque about origine

$\vec{\tau}_0 = \vec{r} \times \vec{F} = \vec{0}$

We known that $\vec{\tau}_0 = \frac{d\vec{L}}{dt} = 0$

Hence Angular momentum will be conserved

Kinetic energy of particle $= \frac{P^2}{2m} = \text{constant}$

constant force is conservative in nature

So, mechanical energy is conserved and potential energy will also conserved.

(Q) $\vec{r}(t) = \alpha \cos\omega t\,\hat{i} + \beta \sin\omega t\,\hat{j}$

velocity of particle at time t.

$\vec{v} = \frac{d\vec{v}}{dt} = -\alpha\,\omega \sin\omega t\,\hat{i} - \beta\omega \cos\omega t\,\hat{j}$

acceleration of particle at time t

$\vec{a} = \frac{d\vec{v}}{dt} = -\alpha\,\omega^2 \cos\omega t\,\hat{i} - \beta\omega^2 \sin\omega t\,\hat{j}$

$= -\omega^2 \left[\alpha \cos\omega t\,\hat{i} + \beta \sin\omega t\,\hat{j}\right]$

$= -\omega^2\,\vec{r}$

$\vec{a} = -\omega^2\,\vec{r}$

Force on a particle $\vec{F} = m\vec{a} = -m\omega^2\vec{r}$

$\rightarrow$ Force depends on position of particle so $\vec{P}$ is not constant

$\rightarrow$ $\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \left(-m\omega^2\vec{r}\right) = \vec{o}$

$\frac{d\vec{L}}{dt} = \vec{o}$

$\vec{L} = \text{constant}$

$\rightarrow$ Nature of force is conservative

So, mechanical energy is conserved

$\Rightarrow$ Kinetic energy $\frac{1}{2}mv^2 = \text{variable}$

Ans. 2, 5

(R) $\vec{r}(t) = \alpha\left(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}\right)$

acceleration of particle,

$$a = \frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}$$

$\rightarrow$ Force on a particle $\vec{F} = m\vec{a} = -m\omega^2\vec{r}$

$\rightarrow$ Force is conservative in nature

So, mechanical energy is conserved.

$\rightarrow$ Kinetic energy is constant.

$\rightarrow$ Potential energy is conserved

$\vec{\tau} = \vec{r} \times \vec{F} = \vec{0},$

$$\frac{d\vec{L}}{dt} = 0$$

Angular momentum is conserved,

Ans. 2, 3, 4, 5

(S) $\vec{r}(t) = \alpha t\hat{i} + \frac{\beta}{2}t^2\,\hat{j}$

Velocity of particle at time t

$$\vec{v} = \frac{d\vec{r}}{dt} = \alpha\,\hat{i} + \beta t\,\hat{j}$$

acceleration of particle at time t

$$\vec{a} = \frac{d\vec{v}}{dt} = \beta\,\hat{j} = \text{constant}$$

$\rightarrow$ Force is conservative in nature

$\rightarrow$ speed of particle $= \sqrt{\alpha^2 + (\beta t)^2}$

Kinetic energy is variable

$\rightarrow$ But mechanical energy is conserved

$\rightarrow$ Potential energy variable

$\rightarrow$ Angular momentum

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \left(\alpha t\,\hat{i} + \frac{\beta}{2}t^2\,\hat{j}\right) \times m\beta\hat{j}$$

$\vec{t} = m\alpha\beta t\hat{k} = \text{variable}$

Ans. (5)