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JEE (ADVANCED) 2018 PAPER-1

[PAPER WITH SOLUTION]

HELD ON SUNDAY 20TH MAY, 2018

PHYSICS

SECTION 1 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: **+4**	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: **+3**	If all the four options are correct but ONLY three options are chosen.
Partial Marks	: **+2**	If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	: **+1**	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	: **0**	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	: **–2**	In all other cases.

- **For example :** If If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

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1. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r)=kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true?

(A) $v=\sqrt{\frac{k}{2m}}R$ (B) $v=\sqrt{\frac{k}{m}}R$ (C) $L=\sqrt{mk}R^2$ (D) $L=\sqrt{\frac{mk}{2}}R^2$

1. **(B, C)**

Given : $v=K\frac{r^2}{2}=\frac{K}{2}r^2$

$\because \quad F_r=-\frac{dv}{dr}=\frac{-K}{2}2r=-Kr$

Now, $|F_r| = \frac{m\nu^2}{R}$

$\Rightarrow \nu = \sqrt{\frac{R\,|F_r|}{m}} = \sqrt{\frac{KrR}{m}}$

$\therefore \quad \nu = \sqrt{\frac{K}{m}}R \rightarrow$ (B)

Also, $L = m\nu R = m\sqrt{\frac{K}{m}}\,R^2$

$= \sqrt{Km}\,R^2 \quad \rightarrow$ (C)

(B, C)

2. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F} = \left(\alpha t\hat{i} + \beta\hat{j}\right)$ is applied on the body, where $\alpha = 1.0\text{Ns}^{-1}$ and $\beta = 1.0\text{N}$. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true?

(A) $|\vec{\tau}| = \frac{1}{3}\text{Nm}$

(B) The torque $\vec{\tau}$ is in the direction of the unit vector + $\hat{k}$

(C) The velocity of the body at t = 1 s is $\vec{v} = \frac{1}{2}\left(\hat{i} + 2\hat{j}\right)\text{m s}^{-1}$

(D) The magnitude of displacement of the body at t = 1 s is 1/6 m

Ans. (A, C)

Sol. Given; m = 1 kg

$\vec{F} = \alpha t\,\vec{i} + B\vec{j}$

Also, $\alpha = 1$ and $\beta = 1$

Clearly, $a_x = \alpha t$

$\Rightarrow \frac{dv_x}{dt} = \alpha t \qquad\qquad \Rightarrow v_x = \frac{t^2}{2}$

$\Rightarrow \int_0^x dx = \frac{1}{2}\int_0^t t^2\,dt$

$\therefore x = \frac{t^3}{6}$

Also, $a_y = \beta = \text{constant}$; $v_y = 0 + \beta t$

$\therefore v_y = t$

$\therefore y = 0 + 0 \times t + \frac{1}{2} \times \beta t^2 \quad = \frac{1}{2}\beta t^2$

As, $\beta = 1$

$\therefore y = \frac{t^2}{2}$

$\therefore \vec{r} =$ position vector of particle at time t about origin O.

$= x\,\vec{i} + y\vec{j} = \frac{t^3}{6}\vec{i} + \frac{t^2}{2}\vec{j}$

Now, $\vec{\tau} = \vec{r} \times \vec{F}$

$= \left(\frac{t^3}{6}\vec{i} + \frac{t^2}{2}\vec{j}\right) \times \left(\alpha t\vec{i} + \beta\vec{j}\right)$

$= \frac{t^3\beta}{6}\vec{K} - \frac{\alpha t^3}{2}\vec{K}$

$= \left(\frac{t^3}{6} - \frac{t^3}{2}\right)\vec{K}$

$= \frac{t^3(1-3)}{6}\vec{K}$

$= \frac{t^3}{3}\left(-\vec{K}\right)$

$|\vec{\tau}|\left(\text{At } t = 1\text{ sec}\right) = \frac{1^3}{2} = \frac{1}{3}\text{N} - \text{m} \quad \rightarrow (\text{A})$

$\vec{v}\left(\text{At } t = 1\text{sec}\right) = \frac{1^2}{2}\vec{i} + 1\vec{j} = \frac{\left(\vec{i} + 2\vec{j}\right)}{2}\frac{\text{m}}{\text{sec}} \rightarrow (\text{C})$

$\because \vec{r}\left(\text{At } t = 1\text{sec}\right) = \frac{1^3}{6}\vec{i} + \frac{1^2}{2} = \frac{1}{6}\vec{i} + \frac{1}{2}\vec{j}$

$\therefore |\vec{r}| = \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{2}\right)^2}$

$= \sqrt{\frac{1}{36} + \frac{1}{4}} = \sqrt{\frac{1+9}{36}}$

$= \frac{\sqrt{10}}{6}\text{m}$

(A, C)

3. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true?

(A) For a given material of the capillary tube, h decreases with increase in r

(B) For a given material of the capillary tube, h is independent of σ

(C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases

(D) h is proportional to contact angle θ

3. **(A), (C)**

$$h = \frac{2\sigma\cos\theta_c}{\rho g r}$$

As, $h \propto \frac{1}{r}$

So, h decreases with increase in r

In a lift going upward with constant acceleration a_0

$g_{eff} = g + a_0$

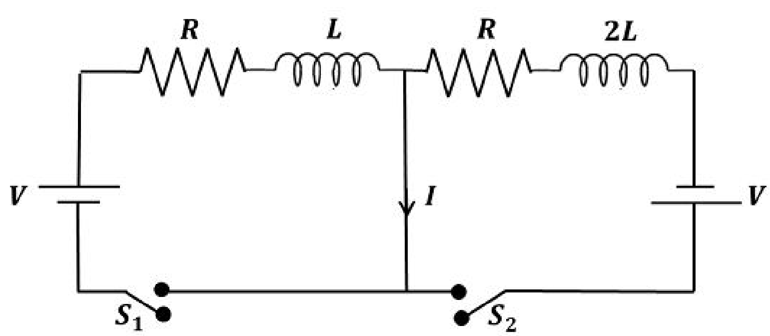
So, $h = \frac{2\sigma\cos\theta_c}{\rho(g + a_0)r}$

So, h decreases if lift accelerates upward

As $h \propto \cos\theta_c$ So option (D) is incorrect

So, option (A) & (C) are correct

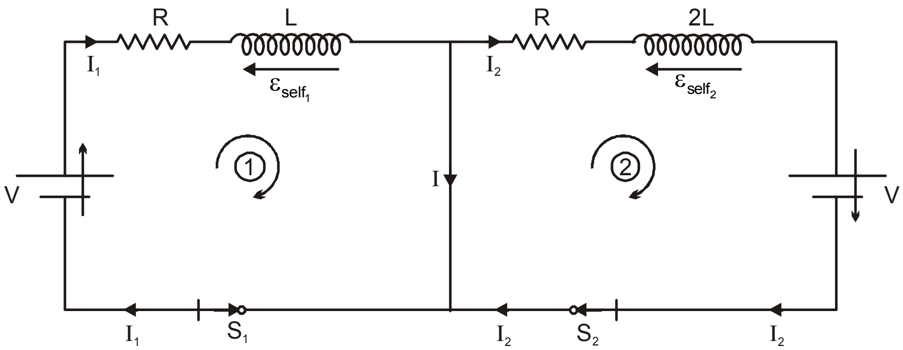
4. In the figure below, the switches S_1 and S_2 are closed simultaneously at t = 0 and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time t = τ. Which of the following statements is (are) true?



(A) $I_{max} = \frac{V}{2R}$ (B) $I_{max} = \frac{V}{4R}$ (C) $\tau = \frac{L}{R}\ln 2$ (D) $\tau = \frac{2L}{R}\ln 2$

4. **(B), (D)**

Sol.



In loop (1), $-V + I_1R + L\dfrac{dI_1}{dt} = 0$ ...(1)

In loop (2), $I_2R + 2L\dfrac{dI_2}{dt} - V = 0$ ...(2)

From (1), $L\dfrac{dI_1}{dt} = V - I_1R = -R\left(I_1 - \dfrac{V}{R}\right)$

$$\Rightarrow \int_0^I \frac{dI_1}{\left(I_1 - \frac{V}{R}\right)} = \frac{-R}{L}\int_0^f dt$$

$$\Rightarrow \ell n\frac{\left(I_1 - \frac{V}{R}\right)}{\left(-\frac{V}{R}\right)} = \frac{-R}{L}t$$

$$\Rightarrow \frac{\left(I_1 - \frac{V}{R}\right)}{\left(\frac{-V}{R}\right)} = e^{-\frac{R}{L}t}$$

$$\therefore \quad I_1 = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

Similarly, $I_2 = \dfrac{V}{R}\left(1 - e^{-\frac{R}{2L}t}\right)$

$$\therefore \quad I = I_1 - I_2 = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t} - \frac{V}{R} + \frac{V}{R}e^{-\frac{R}{2L}t}$$

$\therefore \quad I = \frac{V}{R}\left[e^{-\frac{R}{2L}t} - e^{-\frac{R}{L}t}\right]$

Now, $\frac{dI}{dt} = \frac{V}{R}\left[e^{-\frac{R}{2L}t} \times \left(\frac{-R}{2L}\right) - e^{-\frac{R}{L}t} \times \left(\frac{-R}{L}\right)\right]$

For I to be maximum $\frac{dI}{dt} = 0$

$\Rightarrow \quad \left(e^{\frac{-R}{2L}t}\right) \times \left(\frac{-R}{2L}\right) = e^{-\frac{R}{L}t}\left(-\frac{R}{L}\right)$

$\Rightarrow \quad e^{-\frac{Rt}{2L}} = 2e^{\frac{-Rt}{L}}$

$\Rightarrow \quad \frac{-Rt}{2L} = \ell n2 - \frac{Rt}{L}$

$\Rightarrow \quad \frac{Rt}{L}\left(1 - \frac{1}{2}\right) = \ell n2$

$\Rightarrow \quad \frac{1}{2}\frac{Rt}{L} = \ell n2$

$\therefore \quad t = \frac{2L}{R}\ell n\,(2)$

Now, $I_{max} = \frac{V}{R}\left[e^{\frac{-R}{2L}\frac{2L}{R}\ell n(2)} - e^{-\frac{R}{L}\frac{2L}{R}\ell n(2)}\right]$

$= \frac{V}{R}\left[e^{-\ell n(2)} - e^{-2\ell n(2)}\right]$

$= \frac{V}{R}\left(\frac{1}{2} - \frac{1}{4}\right) \quad = \frac{V}{4R}$

5. Two infinitely long straight wires lie in the xy-plane along the lines x = $\pm$R. The wire located at x = +R carries a constant current I_1 and the wire located at x = –R carries a constant current I_2. A circular loop of radius R is suspended with its centre at (0,0,"3 R) and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the + $\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true?

(A) If $I_1 = I_1$, then $\vec{B}$ **cannot** be equal to zero at the origin (0,0,0)

(B) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin (0,0,0)

(C) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin (0,0,0)

(D) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

Ans. (A), (B), (D)

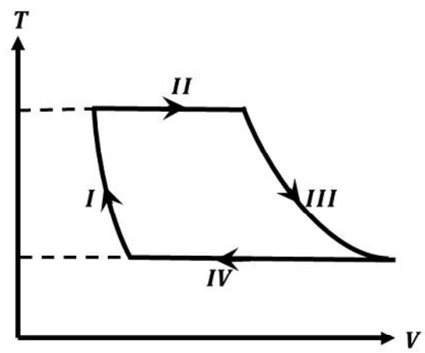
Sol. (A) If $I_1 = I_2$ then $\vec{B}$ at origin will be zero due to two wires but will be non-zero due to ring

(B) If $I_1 > 0$ and $I_2 < 0$ then $\vec{B}$ (at origin) due to two wires in direction opposite to $\vec{B}$ (at origin) due to ring hence $\vec{B}$ can be equal to zero at the origin.

(C) If $I_1 < 0$ and $I_2 > 0$ then $\vec{B}$ due to both wires and ring will be in same direction have $\vec{B}$ can will never be equal to zero at origin.

(D) If $I_1 = I_2$ then the z component at the cnetre of loop is only due to $\vec{B}$ of ring and this value will be $\left(\frac{-\mu_0 I}{2R}\right)$

6. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where **V** is the volume and **T** is the temperature). Which of the statements below is (are) true ?



(A) Process I is an isochoric process

(B) In process II, gas absorbs heat

(C) In process IV, gas releases heat

(D) Processes I and III are **not** isobaric

Ans. (B), (C), (D)

Sol. (A) Isochoric process must be $\perp_r$ to V axis on T – V diagram

(B) During process II, volume is increasing hence w = +ve, Also $\Delta T = 0$

$\therefore \; Q = W + \Delta U = +ve + 0 = +ve$

Thus, gas absorbes heat during process II

(C) During process IV, Volume is decreasing while temperature is constant hence W = –ve and $\Delta U = 0$

$\therefore \; Q = W + \Delta U = -ve + 0 = -ve$

(D) On T–V diagram, isobaric process becomes a straight line passing through origin (but not touching origin) hence I and III are not isobaric

7. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos\omega t\hat{i} + \sin\omega t\hat{j})$, where a is a constant and $\omega = \pi/6\,\text{rad}\,s^{-1}$. If $|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}|$ at time t = τ for the first time, the value of τ, in seconds, is __________.

Ans. **(2)**

Sol. Given : $\vec{A} = a\hat{i} \quad \therefore |\vec{A}| = a$

Also, $\vec{B} = a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}$ and $|\vec{B}| = a$, Further $\omega = \frac{\pi}{6}\frac{\text{rad}}{\text{sec.}}$

$\because \quad \vec{A}.\vec{B} = a^2\cos\omega t$

$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{a^2\cos\omega t}{a^2}$

$\Rightarrow \quad \cos\theta = \cos\omega t$

$\therefore \quad \theta = \omega t$

Given : $|\vec{A}+\vec{B}| = \sqrt{3}\,|\vec{A}-\vec{B}|$

$\Rightarrow \quad \sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{3}\,\sqrt{A^2+B^2-2AB\cos\theta}$

$\Rightarrow \quad A^2+B^2+2AB\cos\theta = 3(A^2+B^2-2AB\cos\theta)$

$\Rightarrow \quad 2A^2+2B^2-8AB\cos\theta = 0$

$\Rightarrow \quad 2a^2+2a^2-8aa\cos\omega t = 0$

$\Rightarrow \quad 4a^2(1-2\cos\omega t) = 0$

$\Rightarrow \quad 1-2\cos\omega t = 0$

$\Rightarrow \quad 2\cos\omega t = 1$

$\Rightarrow \quad \cos\omega t = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow \quad \omega t = \frac{\pi}{3}$

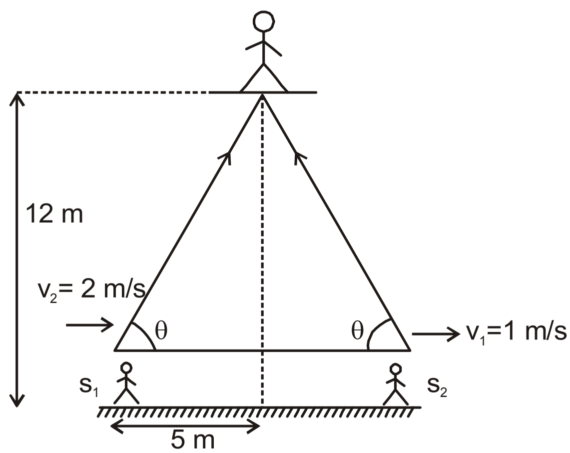
$\therefore \quad t = \frac{\pi}{3\omega} = \frac{\pi\times 6}{3\times\pi} = 2$

8. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 m s^{-1} and the man behind walks at a speed 2.0 m s^{-1}. A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 m s^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is __________.

8. **(5)**

Sol. Here, $\cos\theta = \frac{5}{13}$

Frequency of sound received by S_1, $f_1 = \frac{v}{(v - v_2\cos\theta)} f_0$



So, $f_1 = \frac{330}{\left(330 - \frac{10}{13}\right)} \times 1430$

$= \frac{33 \times 13}{(33 \times 13 - 1)} \times 1430 = 1433.34$ Hz

Frequency of sound received by S_2 is f_2

Where $f_2 = \frac{v}{(v + v_2\cos\theta)} f_0$

$= \frac{330}{\left(330 + \frac{5}{13}\right)} \times 1430$

$= 1428.33$ Hz

So, Beat frequency heard by stationary observer

$\Delta f = f_1 - f_2 = 5$ Hz

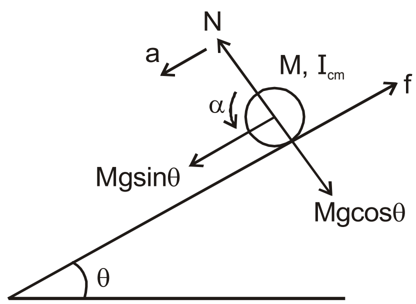
SECTION 2 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place;** e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : **+3** If ONLY the correct numerical value is entered as answer.
  Zero Marks **: 0** In all other cases.

9. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $\left(2-\sqrt{3}\right)/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is __________. Take g = 10 m s^{-2}.

Ans. 0.75

Sol.



For translational motion,

$$Mg\sin\theta - f = Ma_{cm} \quad .....(i)$$

for rotational motion, $f\,R = I_{cm}\,\alpha$

$$\text{So, } f = I_{cm}\frac{a_{cm}}{R^2} \quad ......(ii)$$

From equation (i) & (ii)

$$a_{cm} = \frac{Mg\sin\theta}{\left(M + \frac{I_{cm}}{R^2}\right)}$$

$$\therefore \quad a_{cm} = \frac{g\sin\theta}{\left(1 + \frac{I_{cm}}{MR^2}\right)}$$

For ring, $a_{ring} = \dfrac{g \times \frac{\sqrt{3}}{2}}{(1+1)} = \dfrac{\sqrt{3}\,g}{4}$

For disc, $a_{disc} = \dfrac{g \times \dfrac{\sqrt{3}}{2}}{\left(1+\dfrac{1}{2}\right)} = \dfrac{\sqrt{3}\,g}{3}$

Time taken by ring to reach ground is

$$t_{ring} = \sqrt{\frac{2S}{a_{ring}}} = \sqrt{\frac{2h}{a_{ring}\sin\theta}}$$

$$= \sqrt{\frac{2h}{\dfrac{\sqrt{3}g}{4} \times \dfrac{\sqrt{3}}{2}}} = \sqrt{\frac{16\,h}{3\,g}} = 4\sqrt{\frac{h}{3g}}$$

For disc, $t_{disc} = \sqrt{\dfrac{2h}{\dfrac{\sqrt{3}\,g}{3} \times \dfrac{\sqrt{3}}{2}}} = 2\sqrt{\dfrac{h}{g}}$

So, $t_{ring} - t_{disc} = \sqrt{\dfrac{h}{g}}\left(-2 + \dfrac{4}{\sqrt{3}}\right)$

$$= \frac{2\sqrt{h}}{\sqrt{10}}\left(-1 + \frac{2}{\sqrt{3}}\right) = \frac{2\sqrt{h}}{\sqrt{10}}\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right) \qquad ...(i)$$

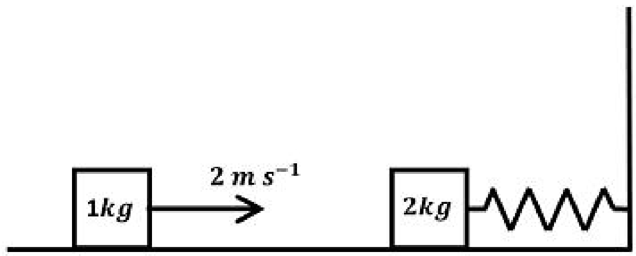
Given , $t_{ring} - t_{disc} = \dfrac{2-\sqrt{3}}{\sqrt{10}}$ ...(ii)

From (i) & (ii), we have

$$\frac{2\sqrt{h}}{\sqrt{10}}\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right) = \frac{2-\sqrt{3}}{\sqrt{10}}$$

$\Rightarrow \sqrt{h} = \dfrac{\sqrt{3}}{2}$, so $h = \dfrac{3}{4}m = 0.75m$

10. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 N m^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 m s^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _________.



Ans. 2.09 m

Sol. Given : $m = 1\,kg, v_o = 2\frac{m}{sec}$ and M = 2kg

Applying law of conservation of linear momentum during collision, we have

$mv_0 + M \times 0 = mv_1 + MV_2$

i.e. $mv_0 = mv_1 + MV_2$ ....(i)

$\Rightarrow \quad 1 \times 2 = 1 \times V_1 + 2 \times V_2$

$\therefore \quad V_1 + 2V_2 = 2$ ....(i)

Also, As collision is elastic

Hence, $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}MV_2^2$

$\Rightarrow \quad 1 \times 2^2 = 1 \times V_1^2 + 2 \times V_2^2$

$\Rightarrow \quad 4 = V_1^2 + 2V_2^2$ ...(ii)

From (i) and (ii), we have

$$4 = v_1^2 + 2\left\{\frac{(2 - v_1)}{2}\right\}^2 = v_1^2 + \frac{(2 - v_1)^2}{2}$$

$\Rightarrow \quad 8 = 2v_1^2 + 4 + v_1^2 - 4v_1$

$\Rightarrow \quad 3v_1^2 - 4v_1 - 4 = 0$

$$\therefore \quad v_1 = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times -4}}{2 \times 3}$$

$= \frac{4 \pm \sqrt{16+48}}{2 \times 3}$

$= \frac{4 \pm 8}{6} = \frac{4+8}{6}, \frac{4-8}{6}$

$= 2, \frac{-2}{3}$

Clearly, $v_1 = \frac{-2}{3}$

Now from (i), $v_2 = \frac{2 - v_1}{2} = \frac{2 + \frac{2}{3}}{2} = 1 + \frac{1}{3} = \frac{4}{3} \frac{m}{sec}$

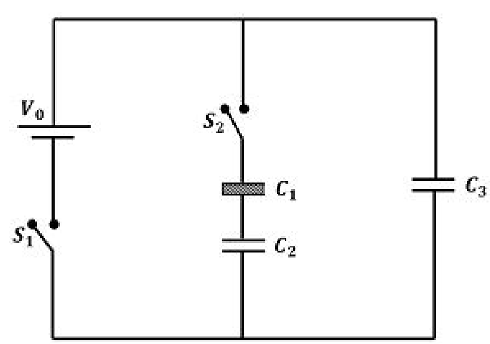
Now, After collision,

Time taken by block of mass M to reach at original position is t'

where $t' = \frac{T}{2} = \frac{2\pi}{2}\sqrt{\frac{m_2}{k}} = \pi\sqrt{\frac{2}{2}} = \pi$ sec

Distane travelled by m_1 during $t' = \frac{2}{3}\pi$ m = 2.09 m

11. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity $\in_r$. The cell electromotive force (emf) V_0 = 8 V. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5 μC. The value of $\in_r$ =____________.



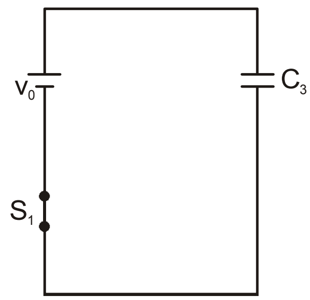
Ans. (1.5)

Sol. Given : $C_1 = C_2 = C_3 = 1\mu F$

$\in_r = ?$

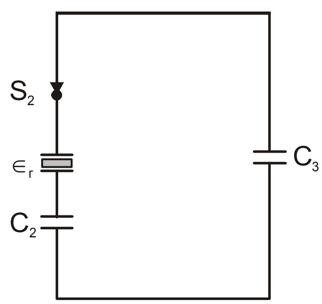
$v_0 = 8$ volt

First phenomena, S_1 is closed and S_2 is kept open



$Q_3 = C_3 V_0 = 1 \times 10^{-6} \times 8 = 8\mu C$

Second phenomena, Now S_1 is opened and S_2 is closed



Given : Charge on C_3 is $5\mu C$ in equilibrium

Thus charge on C_1 and C_2 will be $3\mu C$

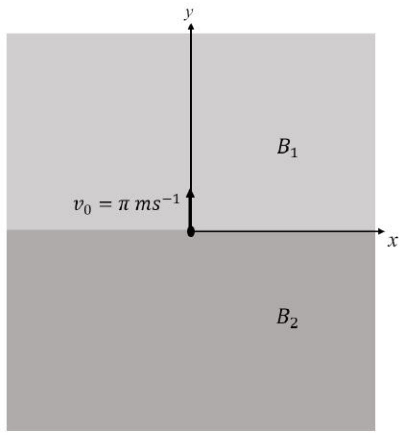
$$\Rightarrow \quad \frac{3}{\in_r C_1} + \frac{3}{C_2} = \frac{5}{C_3}$$

$$\Rightarrow \quad \frac{3}{\in_r} + 3 = 5$$

$$\Rightarrow \quad \frac{3}{\in_r} = 2$$

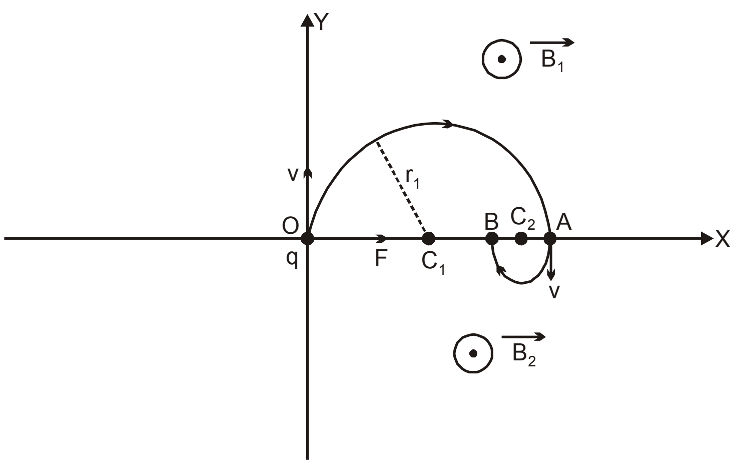
$$\therefore \quad \in_r = \frac{3}{2} = 1.5$$

12. In the xy-plane, the region $y > 0$ has a uniform magnetic field $B_1\hat{k}$ and the region $y < 0$ has another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi$ m s^{-1} at $t = 0$, as shown in the figure. Neglect gravity in this problem. Let $t = T$ be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in m s^{-1}, along the x-axis in the time interval T is __________.



Ans. **2.00 m/s**

Sol.



$$r_1 = \frac{mv}{qB_1} \ \& \ r_2 = \frac{mv}{qB_2} = \frac{r_1}{4}$$

time to go from $O \rightarrow A$ $t_{0 \rightarrow A} = \frac{\pi r_1}{v_0} = r_1$ $\quad$ [Given $v_0 = \pi$ m/s]

time to go from $A \rightarrow B$ $t_{A \rightarrow B} = \frac{\pi r_2}{v_0} = r_2$

So total time

$$t = r_1 + r_2 = \frac{5r_1}{4}$$

So avg. speed along X-axis (Considering projection of particle on x-axis)

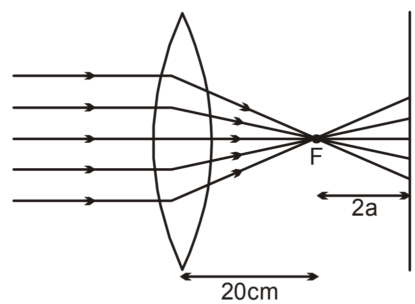
$$(v_{avg})_x = \frac{2r_1 + 2r_2}{t}$$

$$= \frac{2(r_1 + r_2)}{(r_1 + r_2)} = 2 \text{ m/s}$$

13. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is __________.

Ans. 130 kw/m²

Sol. F acts as point source

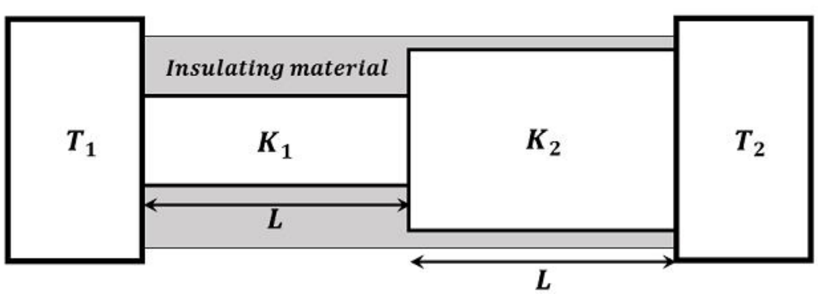


So $I_1 \propto \frac{1}{r^2}$

So $I_2 = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{20}{2}\right)^2$

$I_2 = 100\, I_1 = 130 \text{ kW/m}^2$

14. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures T_1 = 300 K and T_2 = 100 K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then K_1/K_2= __________.



Ans. 4

Sol. Thormal resistane of rod (1)

$$R_1 = \frac{L}{K_1 A_1}$$

Thermoal resistance of rod (2)

$$R_2 = \frac{L}{K_2A_2}$$

for rod (1) $i_1 = \frac{T_1 - T_i}{R_1} = \frac{300 - 200}{R_1} = \frac{100}{R_1}$

As rods are in series so $i_1 = i_2$

$$\frac{100}{R_1} = \frac{100}{R_2} \qquad \text{So, } R_1 = R_2$$

i.e. $\frac{L}{K_1A_1} = \frac{L}{K_2A_2}$

i.e. $K_1\pi r^2 = K_2 \times 4\pi r^2$ So, $\frac{K_1}{K_2} = 4$

SECTION 3 (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+3** If ONLY the correct option is chosen.
Zero Marks **: 0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : **–1** In all other cases.

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while $[\epsilon_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

(There are two questions based on PARAGRAPH "X", the question given below is one of them)

15. The relation between $[E]$ and $[B]$ is

(A) [E] = [B] [L] [T]
(B) [E] = [B] $[L]^{-1}$ [T]
(C) [E] = [B] [L] $[T]^{-1}$
(D) [E] = [B] $[L]^{-1}$ $[T]^{-1}$

Ans. **(C)**

Sol. As $\frac{E}{B} = C$ where $C \to$ speed of light

So, E = BC

$[E] = [B]\,[L]\,[T]^{-1}$

So, option (C) is correct.

16. The relation between $[\epsilon_0]$ and $[\mu_0]$ is

(A) $[\mu_0] = [\epsilon_0]\ [L]^2\ [T]^{-2}$

(B) $[\mu_0] = [\epsilon_0]\ [L]^{-2}\ [T]^2$

(C) $[\mu_0] = [\epsilon_0]^{-1}\ [L]^2\ [T]^{-2}$

(D) $[\mu_0] = [\epsilon_0]^{-1}\ [L]^{-2}\ [T]^2$

Ans. **(D)**

Sol. Again $C = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

So, $\mu_0 \varepsilon_0 = \dfrac{1}{C^2}$

$[\mu_0] = [\varepsilon_0]^{-1}\ [L]^{-2}\ [T]^2$

option (D) is correct

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = x/y$. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta x}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \dfrac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\Delta x/x << 1,\ \Delta y/y << 1.$. Therefore, the higher powers of these quantities are neglected.

(There are two questions based on PARAGRAPH "A", the question given below is one of them)

17. Consider the ratio $r = \dfrac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is $\Delta a\,(\Delta a/a << 1)$, then what is the error Δr in determining r?

(A) $\dfrac{\Delta a}{(1+a)^2}$

(B) $\dfrac{2\Delta a}{(1+a)^2}$

(C) $\dfrac{2\Delta a}{(1-a^2)}$

(D) $\dfrac{2a\Delta a}{(1-a^2)}$

17. **(B)**

Sol. As $r = \frac{(1-a)}{(1+a)}$

So, $\frac{\Delta r}{r} = \frac{\Delta(1-a)}{(1-a)} + \frac{\Delta(1-a)}{(1+a)} = \frac{\Delta a}{(1-a)} + \frac{\Delta a}{(1+a)}$

$$\frac{\Delta r}{r} = \frac{2\Delta a}{(1-a^2)}$$

So, $\Delta r = \frac{2\Delta a}{(1-a^2)} \times \frac{(1-a)}{(1+a)} = \frac{2\Delta a}{(1+a)^2}$

option (B) is correct.

18. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For |x| <<1, ln(1 + x) = x up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is

(A) 0.04 (B) 0.03 (C) 0.02 (D) 0.01

18. (C)

Sol. As, $N = N_0 e^{-\lambda t}$

So, $\ell n\, N = \ell n\, N_0 - \lambda t$

Diff. w.r.t. λ

$$\frac{1}{N} \times \frac{dN}{d\lambda} = 0 - t$$

So, $d\lambda = \frac{dN}{Nt} = \frac{40}{2000 \times 1} = 0.02$

Option C is correct.