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This booklet contains 24 printed pages.

Test Booklet Code

A

PAPER-1 : PHYSICS, CHEMISTRY & MATHEMATICS

Do not open this Test Booklet until you are asked to do so.

Read carefully the Instructions on the Back Cover of this Test Booklet.

Important Instructions :

1. Immediately fill in the particulars on this page of the Test Booklet with *Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.*

2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions. The maximum marks are **360.**

5. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** and having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for correct response.

6. *Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*

7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

8. Use ***Blue/Black Ball Point pen only*** for writing particulars/marking responses on ***Side-1*** and ***Side-2*** of the Answer Sheet. ***Use of pencil is strictly prohibited.***

9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.

10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is give at the bottom of each page and in **three** pages (Pages **21 - 23**) at the end of the booklet.

11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***

12. The CODE for the Booklet is **C.** Make sure that the CODE printed on **Side-2** of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.

13. **Do not fold or make any stray marks on the Answer Sheet.**

Name of the Candidate (in Capital Letters): ______________________________

Roll Number : In figures | | | | | | | | |

: in words ______________________________

Examination Centre Number : | | | | | | |

Name of Examination Centre (in Capital letters) :______________________________

Candidate's Signature :________________ 1. Invigilator's Signature : ________________

2. Invigilator's Signature : ________________

PART-C : MATHEMATICS

61. Two sets A and B are as under:

$A=\{(a,b)\in R\times R: |a-5|<1 \text{ and } |b-5|<1\}$;

$B=\{(a,b)\in R\times R: 4(a-6)^2+9(b-5)^2\le 36\}$. Then:

(1) $B\subset A$ (2) $A\subset B$

(3) $A\cap B=\phi$ (an empty set) (4) neither $A\subset B$ nor $B\subset A$

Ans. (2)

Sol.

$|a-5|<1 \Rightarrow -1<a-5<1$

$\Rightarrow -2<a-6<0$

$\Rightarrow 0<(a-6)^2<4$ ....(i)

$|b-5|<1$

$\Rightarrow 0\le(b-5)^2<1$ ....(ii)

From (i) and (ii)

$0<4(a-6)^2+9(b-5)^2<25$

$\Rightarrow A\subset B$

62. Let $S=\{x\in R: x\ge 0 \text{ and } 2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0\}$. Then S:

(1) is an empty set (2) contains exactly one element

(3) contains exactly two elements (4) contains exactly four elements

Ans. (3)

Sol.

$2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0$

Let $\sqrt{x}=t$

$2|t-3|+t(t-6)+6=0$

Case:I

$t\ge 3$

$\Rightarrow 2t-6+t^2-6t+6=0$

$\Rightarrow t^2-4t=0 \Rightarrow t(t-4)=0$

$t=0,4$

then $\sqrt{x}=4\Rightarrow x=16$

Case:II

$0 \le t < 3$

$\Rightarrow 6 - 2t + t^2 - 6t + 6 = 0 \;\Rightarrow t^2 - 8t + 12 = 0$

$(t-6)(t-2) = 0$

$t = 2, 6$, where t = 2 is valid

then $\sqrt{x} = 2 \;\Rightarrow\; x = 4$

63. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

(1) –1 (2) 0 (3) 1 (4) 2

Ans. (3)

Sol.

$x^2 - x + 1 = 0 \Rightarrow x = -\omega, -\omega^2$

Let $\alpha = -\omega$ and $\beta = -\omega^2$

$\therefore \alpha^{101} + \beta^{107} = (-\omega)^{101} + (-\omega^2)^{107} = -(\omega^{101} + \omega^{214}) = -(\omega^2 + \omega) = -(-1) = 1$

64. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to

(1) (– 4, – 5) (2) (– 4, 3) (3) (– 4, 5) (4) (4, 5)

Ans. (3)

Sol.

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

Apply $C_1 \to C_1 + C_2 + C_3$,

$$= (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x-4 & 2x \\ 1 & 2x & x-4 \end{vmatrix}$$

Apply $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$,

$$= (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x-4 & 0 \\ 0 & 0 & -x-4 \end{vmatrix}$$

$= (5x-4)(x+4)^2 \Rightarrow A = -4, B = 5$

65. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x,y,z), then $\frac{xz}{y^2}$ is equal to

(1) – 10 (2) 10 (3) – 30 (4) 30

Ans. (2)

Sol.

According to given condition,

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0 \Rightarrow 44 - 4k = 0 \Rightarrow k = 11$$

therefore, the given system of equation is

$x + 11y + 3z = 0$ ....(i)

$3x + 11y - 2z = 0$ ....(ii)

$2x + 4y - 3z = 0$ ....(iii)

From (i) and (ii) $2x - 5z = 0$ ....(iv)

& from (i) and (iii) $3x + 15y = 0$ ....(v)

$$\therefore \frac{xz}{y^2} = \frac{x \cdot \frac{2x}{5}}{\frac{x^2}{25}} = 10$$

66. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is

(1) at least 1000 (2) less than 500

(3) at least 500 but less than 750 (4) at least 750 but less than 1000

Ans. (1)

Sol.

Required number of ways = ${}^6C_4 \times {}^3C_1 \times 4! = 1080$

67. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5, (x > 1)$ is

(1) – 1 (2) 0 (3) 1 (4) 2

Ans. (4)

Sol.

$$\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5 = 2\left\{x^5 + 10x^3\left(x^3 - 1\right) + 5x\left(x^3 - 1\right)^2\right\}$$

$$= 2\left\{x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x\right\}$$

Hence, the required sum of coefficients is 2

68. Let $a_1, a_2, a_3, ...,a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to

(1) 66 (2) 68 (3) 34 (4) 33

Ans. **(3)**

Sol.

Let the common difference be d

$$\because \sum_{k=0}^{12} a_{4k+1} = 416$$

$$\therefore \frac{13}{2}(a_1 + a_{49}) = 416$$

$$\Rightarrow a_1 + a_{49} = 64$$

$$\Rightarrow 2a_1 + 48d = 64 \quad ......(i)$$

further, $a_9 + a_{43} = 66$

$$\Rightarrow 2a_1 + 50d = 66 \quad ......(ii)$$

From (i) and (ii)

$a_1 = 8$ and d = 1

Now, $a_1^2 + a_2^2 + ... + a_{17}^2 = 140$ m $\qquad \therefore m = \dfrac{a_1^2 + a_2^2 + ... + a_{17}^2}{140}$

$$= \frac{8^2 + 9^2 + ... + 24^2}{140} = \frac{(1^2 + 2^2 + ... + 24^2) - (1^2 + 2^2 + ... + 7^2)}{140}$$

$$\frac{1}{140}\left\{\frac{24 \cdot 25 \cdot 49}{6} - \frac{7 \cdot 8 \cdot 15}{6}\right\} = 34$$

69. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + ...$ .If $B - 2A = 100\lambda$, then λ is equal to

(1) 232 (2) 248 (3) 464 (4) 496

Ans. **(2)**

Sol.

$$A = 1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + ... + 2 \cdot 20^2$$

$$= (1 + 2^2 + 3^2 + ... + 20^2) + (2^2 + 4^2 + ... + 20^2)$$

$$= \frac{20(21)(41)}{6} + 2^2\left(\frac{10(11)21}{6}\right)$$

$$= 2870 + 1540 = 4410$$

$B = 1^2 + 2^2 + ... + 40^2 + \left(2^2 + 4^2 + ... + 40^2\right)$

$= \frac{40 \times 41 \times 81}{6} + 4 \times 2870$

= 22140 + 11480 = 33620

B – 2A = 33620 – 8820 = 24800

B – 2A = 100λ

$\Rightarrow 24800 = 100\lambda \qquad \therefore \lambda = 248$

70. For each $t \in R$, let [t] be the greatest integer less than or equal to t. Then $\lim_{x \to 0+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$

(1) is equal to 0
(2) is equal to 15
(3) is equal to 120
(4) does not exist (in R)

Ans. (3)

Sol.

$$\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \left[\frac{3}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$$

$$= \lim_{x \to 0^+} x\left(\frac{15 \times 16}{2x}\right) - \lim_{x \to 0^+} x\left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + ... + \left\{\frac{15}{x}\right\}\right)$$

= 120 – 0 = 120

71. Let $S = \left\{t \in R : f(x) = |x - \pi| \cdot \left(e^{|x|} - 1\right)\sin|x| \text{ is not differentiable at } t\right\}$. Then the set S is equal to :

(1) ϕ (an empty set)
(2) $\{0\}$
(3) $\{\pi\}$
(4) $\{0, \pi\}$

Ans. (1)

Sol. $f(x) = |x - \pi|(e^{|x|} - 1)\sin|x|$

$|x - \pi| = 0$ at $x = \pi$

and $|x| = 0$ at $x = 0$

But at x = 0 both $e^{|x|} - 1$ and $\sin|x|$ are zero.

and at $x = \pi$ both $|x - \pi|$ and $\sin|x|$ are zero

$\therefore$ f is differentiable in R.

72. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$, intersect each other at right angles, then the value of b is :

(1) 6 (2) $\frac{7}{2}$ (3) 4 (4) $\frac{9}{2}$

Ans. (4)

Sol. $y^2 = 6x$

$\Rightarrow 2y\frac{dy}{dx} = 6$

$\Rightarrow \frac{dy}{dx} = \frac{3}{y}$ ... (i)

$9x^2 + by^2 = 16$

$\Rightarrow 18x + 2by\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-9x}{by}$ ...(ii)

Since curves intersect at right angles

$\therefore$ product of slopes = – 1

$\Rightarrow \frac{-27x}{by^2} = -1$

$\Rightarrow 27x = by^2$

$\Rightarrow 27x = 6bx$ $[\because y^2 = 6x]$

$(27 - 6b)x = 0$

$\Rightarrow 6b = 27$ $(x \neq 0)$

$b = \frac{9}{2}$

73. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}, x \in R - \{-1,0,1\}$. If $h(x) = \frac{f(x)}{g(x)}$,then local minimum value of h(x) is :

(1) 3 (2) -3 (3) $-2\sqrt{2}$ (4) $2\sqrt{2}$

Ans. (4)

Sol. $f(x) = x^2 + \frac{1}{x^2}$ & $g(x) = x - \frac{1}{x}$

$$h(x)=\frac{f(x)}{g(x)}=\frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}=\frac{\left(x-\frac{1}{x}\right)^2+2}{x-\frac{1}{x}}=\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}$$

Let $x-\frac{1}{x}=t$

$\therefore k(t)=t+\frac{2}{t},\ t\in R$ which attains minimum value for $t=\sqrt{2}$

Hence, local minimum value of $h(x)=2\sqrt{2}$

74. The integral $\int\frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$ is equal to :

(1) $\frac{1}{3(1+\tan^3 x)}+C$ (2) $\frac{-1}{3(1+\tan^3 x)}+C$ (3) $\frac{1}{1+\cot^3 x}+C$ (4) $\frac{-1}{1+\cot^3 x}+C$

(where C is a constant of integration)

Ans. (2)

Sol. $\int\frac{\sin^2 x\cos^2 x}{\left(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x\right)^2}\ dx$

$=\int\frac{\sin^2 x\cos^2 x}{(\sin^2 x(\sin^3 x+\cos^3 x)+\cos^2 x(\sin^3 x+\cos^3 x))^2}dx$

$=\int\frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx$

$=\int\frac{\tan^2 x\sec^2 x}{(\tan^3 x+1)^2}dx$

$\begin{pmatrix}\text{put } \tan^3 x+1=z\\ \therefore 3\tan^2 x\sec^2 x\,dx=dz\end{pmatrix}$

$=\frac{1}{3}\int\frac{dz}{z^2}=-\frac{1}{3z}+c$

$=-\frac{1}{3(1+\tan^3 x)}+c$

75. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}dx$ is :

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{2}$ (3) 4π (4) $\frac{\pi}{4}$

Ans. (4)

Sol. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{2^x+1}dx$

$= \int_{0}^{\frac{\pi}{2}} \sin^2 x dx = \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$

76. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$, and $\alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and y = 0, is :

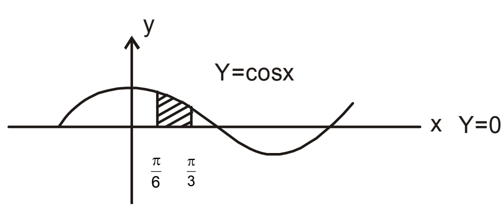
(1) $\frac{1}{2}\left(\sqrt{3}-1\right)$ (2) $\frac{1}{2}\left(\sqrt{3}+1\right)$ (3) $\frac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$ (4) $\frac{1}{2}\left(\sqrt{2}-1\right)$

Ans. (1)

Sol. Solving $18x^2 - 9\pi x + \pi^2 = 0$ we get,

$\alpha = \frac{\pi}{6}$ & $\beta = \frac{\pi}{3}$ $\left(\because \alpha < \beta\right)$

Now, $g(f(x)) = g(\sqrt{x}) = \cos(\sqrt{x})^2 = \cos x$



$\therefore$ Area bounded by $g\left(f(x)\right)$, y = 0, $x = \alpha$, $x = \beta$ is

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x dx = [\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \frac{\sqrt{3}-1}{2}$

77. Let $y = y(x)$ be the solution of the differential eqaution $\sin x \frac{dy}{dx} + y\cos x = 4x, x \in (0,\pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $\frac{4}{9\sqrt{3}}\pi^2$ (2) $\frac{-8}{9\sqrt{3}}\pi^2$ (3) $-\frac{8}{9}\pi^2$ (4) $-\frac{4}{9}\pi^2$

Ans. (3)

Sol. Given differential equation is $\frac{d}{dx}(y\sin x) = 4x$

Solution is : $y \cdot \sin x = \int 4x dx = \frac{4x^2}{2} + c$

$y\sin x = 2x^2 + c$

$c = -\frac{\pi^2}{2}$ $\left(\text{at } x = \frac{\pi}{2}, y = 0\right)$

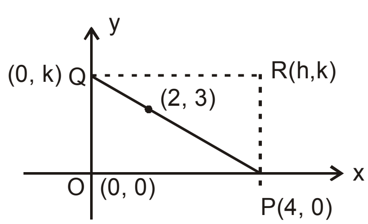
$\therefore\ y\left(\frac{\pi}{6}\right) = -\frac{8}{9}\pi^2$

78. A straight line through a fixed point (2, 3) intersects the coordiante axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :

(1) $3x + 2y = 6$ (2) $2x + 3y = xy$ (3) $3x + 2y = xy$ (4) $3x + 2y = 6xy$

Ans. (3)

Sol.



Equation of QP :

$\frac{x}{h} + \frac{y}{k} = 1$

It passes through (2, 3)

$\frac{2}{h} + \frac{3}{k} = 1$

2k + 3h = hk

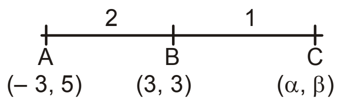
hence Locus of R is : $2y + 3x = xy$.

79. Let the orthocentre and centroid of a triangle be A(– 3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is :

(1) $\sqrt{10}$ (2) $2\sqrt{10}$ (3) $3\sqrt{\frac{5}{2}}$ (4) $\frac{3\sqrt{5}}{2}$

Ans. **(3)**

Sol.



radius $= \frac{AC}{2} = \frac{1}{2}\cdot\frac{3}{2}AB,$ $\left(\because AB = \frac{2}{3}AC\right)$

$= \frac{3}{4}\sqrt{(-3-3)^2 + (5-3)^2} = 3\sqrt{\frac{5}{2}}$

80. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is :

(1) 195 (2)185 (3) 85 (4) 95

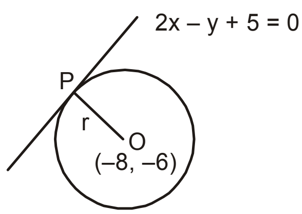
Ans. **(4)**

Sol. tangent at (1, 7) to the curve $x^2 = y - 6$ is

$x \cdot 1 = \frac{1}{2}(y+7) - 6$

i.e $2x - y + 5 = 0$,

It touches $x^2 + y^2 + 16x + 12y + c = 0$



So, OP = r

$\Rightarrow \frac{|-16+6+5|}{\sqrt{4+1}} = \sqrt{64+36-c}$

$\Rightarrow \sqrt{5} = \sqrt{100-c}$

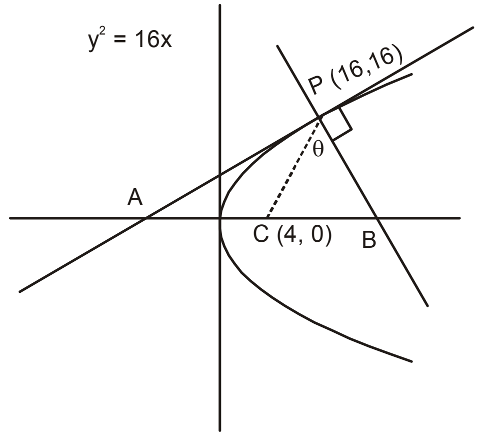
$\Rightarrow c = 95$

81. Tangent and normal are drawn at P(16,16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the point P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is

(1) 1/2 (2) 2 (3) 3 (4) 4

Ans. **(2)**

Sol. Centre of circle passing through, P,A,B is focus of parabola then C = (4, 0).



Tangent at P(16,16) : T = 0 16 y = 8(x + 16)

slope of normal PB is = –2

slope of PC = $\frac{16}{12} = \frac{4}{3}$

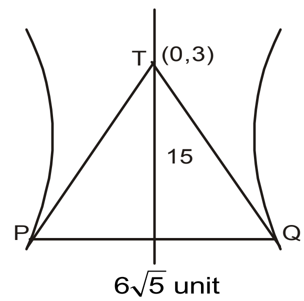
$$\tan\theta = \left|\frac{-2-\frac{4}{3}}{1+(-2)\cdot\frac{4}{3}}\right| = 2$$

82. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0,3) then the area (in sq. units) of ΔPTQ is

(1) $45\sqrt{5}$ (2) $54\sqrt{3}$ (3) $60\sqrt{3}$ (4) $36\sqrt{5}$

Ans. **(1)**

Sol. Chord of contact of tangents from (0, 3) is



$4x \cdot 0 - y \cdot 3 = 36$

i.e. y = -12

$\therefore$ P,Q are given by

$\left(3\sqrt{5}, -12\right)$ and $\left(-3\sqrt{5}, -12\right)$

$\therefore (\Delta TPQ) = \frac{1}{2} \times 15 \times 6\sqrt{5} = 45\sqrt{5}$ sq.units

83. If L_1 is the line of intersection of planes 2x – 2y + 3z – 2 = 0, x – y + z + 1 = 0 and L_2 is the line of intersection of the planes x + 2y – z – 3 = 0, 3x – y + 2z – 1 = 0, then the distance of the origin from the plane, containing the lines L_1 and L_2 is

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

Ans. **(2)**

Sol. Plane containing line L_1 is 2x – 2y + 3z – 2 + α (x – y + z + 1) = 0

$\Rightarrow (\alpha+2)x - (\alpha+2)y + (\alpha+3)z + \alpha - 2 = 0$ ......(i)

Direction ratios of L_2 are (3, –5, –7) and L_2 lies on the plane in equation (i)

$\Rightarrow 3(\alpha+2) + 5(\alpha+2) - 7(\alpha+3) = 0 \Rightarrow \alpha = 5$

Hence the required plane is 7x – 7y + 8z +3 = 0

Now, distance from origin of the plane is

$$= \frac{3}{\sqrt{49+49+64}} = \frac{1}{3\sqrt{2}}$$

84. The length of the projection of the line segment joining the points (5, –1,4) and (4, –1, 3) on the plane, x + y + z = 7 is

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\sqrt{\frac{2}{3}}$

Ans. **(4)**

Sol. Foot of perpendicular from (5, -1, 4) is given by

$$\frac{x-5}{1}=\frac{y+1}{1}=\frac{z-4}{1}=\frac{-(5-1+4-7)}{3}=-\frac{1}{3}$$

i.e. $\left(\frac{14}{3}-\frac{4}{3},\frac{11}{3}\right)=A$ (say)

Foot of perpendicular from (4, -1, 3) is given by

$$\frac{x-4}{1}=\frac{y+1}{1}=\frac{z-3}{1}=\frac{-(4-1+3-7)}{3}=\frac{1}{3}$$

i.e. $\left(\frac{13}{3},-\frac{2}{3},\frac{10}{3}\right)=B$ (say)

$\therefore$ Req. length = AB $=\sqrt{\frac{1}{9}+\frac{4}{9}+\frac{1}{9}}=\sqrt{\frac{2}{3}}$

85. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b}=24$, then $|\vec{u}|^2$ is equal to

(1) 336 (2) 315 (3) 256 (4) 84

Ans. **(1)**

Sol. $\vec{u}=x\vec{a}+y\vec{b}$

$\vec{u}\cdot\vec{a}=0\Rightarrow x|\vec{a}|^2+y\vec{b}\cdot\vec{a}=0$

$\Rightarrow x\cdot 14+2y=0 \Rightarrow y+7x=0$ ...(1)

$\vec{u}\cdot\vec{b}=24\Rightarrow x(\vec{a}\cdot\vec{b})+y(\vec{b})^2=24$

$\Rightarrow 2x+2y=24 \Rightarrow x+y=12$ ...(2)

From (1) and (2), x = –2, y = 14

$\therefore \vec{u}=-2\vec{a}+14\vec{b} = (14\hat{j}+14\hat{k})-(4\hat{i}+6\hat{j}-2\hat{k})=-4\hat{i}+8\hat{j}+16\hat{k}$

$|\vec{u}|^2=16+64+256=336$

86. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is

(1) $\frac{3}{10}$ (2) $\frac{2}{5}$ (3) $\frac{1}{5}$ (4) $\frac{3}{4}$

Ans. **(2)**

Sol. Let R : event that a red ball is drawn, B : event that a black ball is drawn'

$\therefore$ Required probability $= P(R)P(R|R) + P(B)P(R|B) = \frac{4}{10}\times\frac{6}{12}+\frac{6}{10}\times\frac{4}{12}=\frac{2}{5}$

87. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45,$ then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is

(1) 9 (2) 4 (3) 2 (4) 3

Ans. **(3)**

Sol. $\because$ S.D of a series does not change, if each term is decreased by the same quanity,

$$\therefore \text{ the S.D } = \sqrt{\frac{\sum_{i=1}^{9}(x_i-5)^2}{9}-\left(\frac{\sum_{i=1}^{9}(x_i-5)}{9}\right)^2} = \sqrt{\frac{45}{9}-\left(\frac{9}{9}\right)^2} = 2$$

88. If sum of all the solutions of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0,\pi]$ is $k\pi$, then k is equal to

(1) $\frac{2}{3}$ (2) $\frac{13}{9}$ (3) $\frac{8}{9}$ (4) $\frac{20}{9}$

Ans. **(2)**

Sol. $4\cos x\left\{2\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-1\right\}=1$

$$\Rightarrow 4\cos x\left\{\left(\cos\frac{\pi}{3}+\cos 2x\right)-1\right\}=1 \Rightarrow 4\cos x\left(\cos 2x-\frac{1}{2}\right)=1$$

$$\Rightarrow 4\cos x\left(2\cos^2 x-1-\frac{1}{2}\right)=1 \Rightarrow 8\cos^3 x-6\cos x=1$$

$$\Rightarrow 4\cos^3 x-3\cos x=\frac{1}{2} \Rightarrow \cos 3x=\frac{1}{2} \Rightarrow 3x=\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3}$$

89. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45^o, 30^o and 30^o, then the height of the tower (in m) is

(1) 100 (2) 50 (3) $100\sqrt{3}$ (4) $50\sqrt{2}$

Ans. (1)

Sol. Let M is the mid point of QR. and the length of tower is t.

Now, $PM = \sqrt{(200)^2 - (\ell/2)^2}$

$\tan 45° = \frac{MT}{PM}$

$\therefore t = \sqrt{(200)^2 - (\ell/2)^2}$ ....(1)

Also,

$\tan 30° = \frac{TM}{QM} \Rightarrow \frac{1}{\sqrt{3}} = \frac{TM}{QM}$

$\frac{1}{\sqrt{3}} = \frac{t}{\ell/2}$ $\therefore t = \frac{\ell}{2\sqrt{3}}$ ....(2)

From (1) and (2)

$(200)^2 - (\ell/2)^2 = \left(\frac{\ell}{2\sqrt{3}}\right) \Rightarrow (200)^2 = \frac{\ell^2}{3}$

$\therefore \ell = 200\sqrt{3}$

$\therefore$ length of tower $\Rightarrow t = \frac{200\sqrt{3}}{2\sqrt{3}} = 100\,m$

90. The Boolen expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to

(1) ~p (2) p (3) q (4) ~q

Ans. (1)

Sol. $\sim(p \vee q) \vee (\sim p \wedge q)$

$\equiv \sim(p \vee q) \wedge (p \vee \sim q)$ $\equiv \sim \{p \vee (q \wedge \sim q)\}$ $\equiv \sim \{p \vee F\}$ $\equiv \sim p$